Institute for Advanced Study

Multi-planetary systems, Saturn's Rings and the collisional N-body code REBOUND

Hanno Rein @ McGill, November 2011

Planet formation

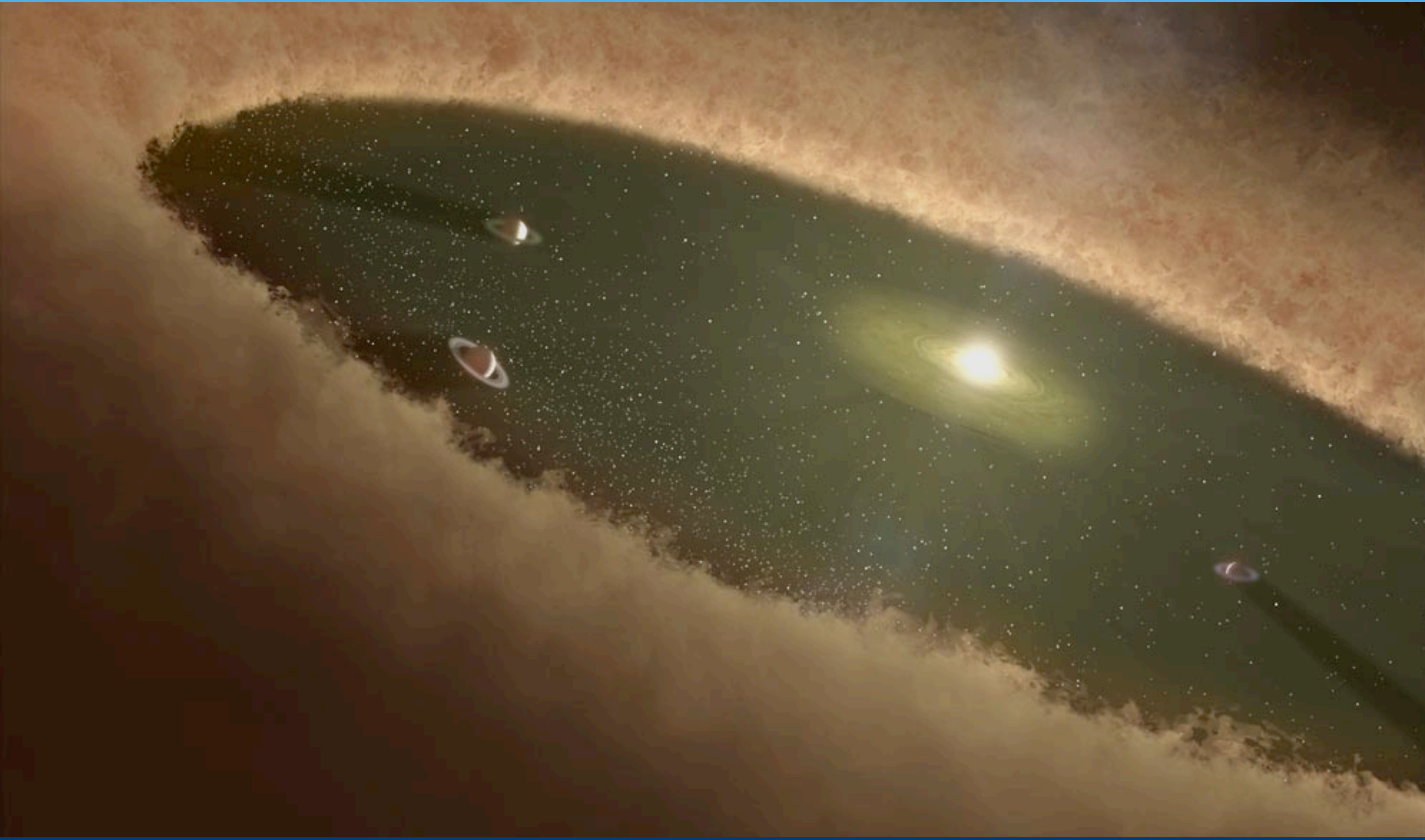


Image credit: NASA/JPL-Caltech

Migration in a non-turbulent disc

Gap opening criteria

Disc scale height

Stellar mass

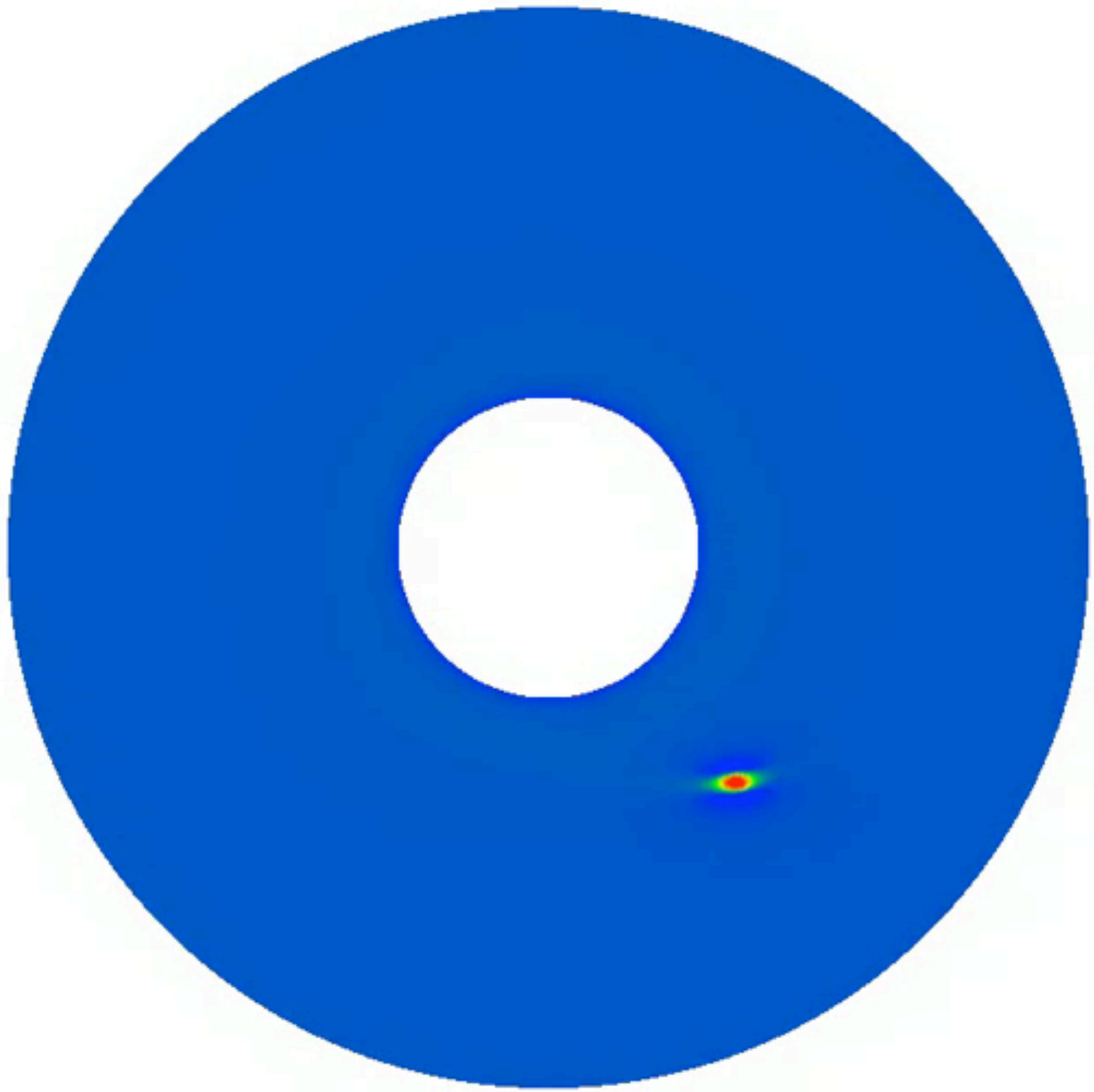
$$\frac{3}{4}\frac{H}{R_{\mathrm{Hill}}} + \frac{50 M_*}{M_p \mathcal{R}} \leq 1$$

Planet mass

Viscosity

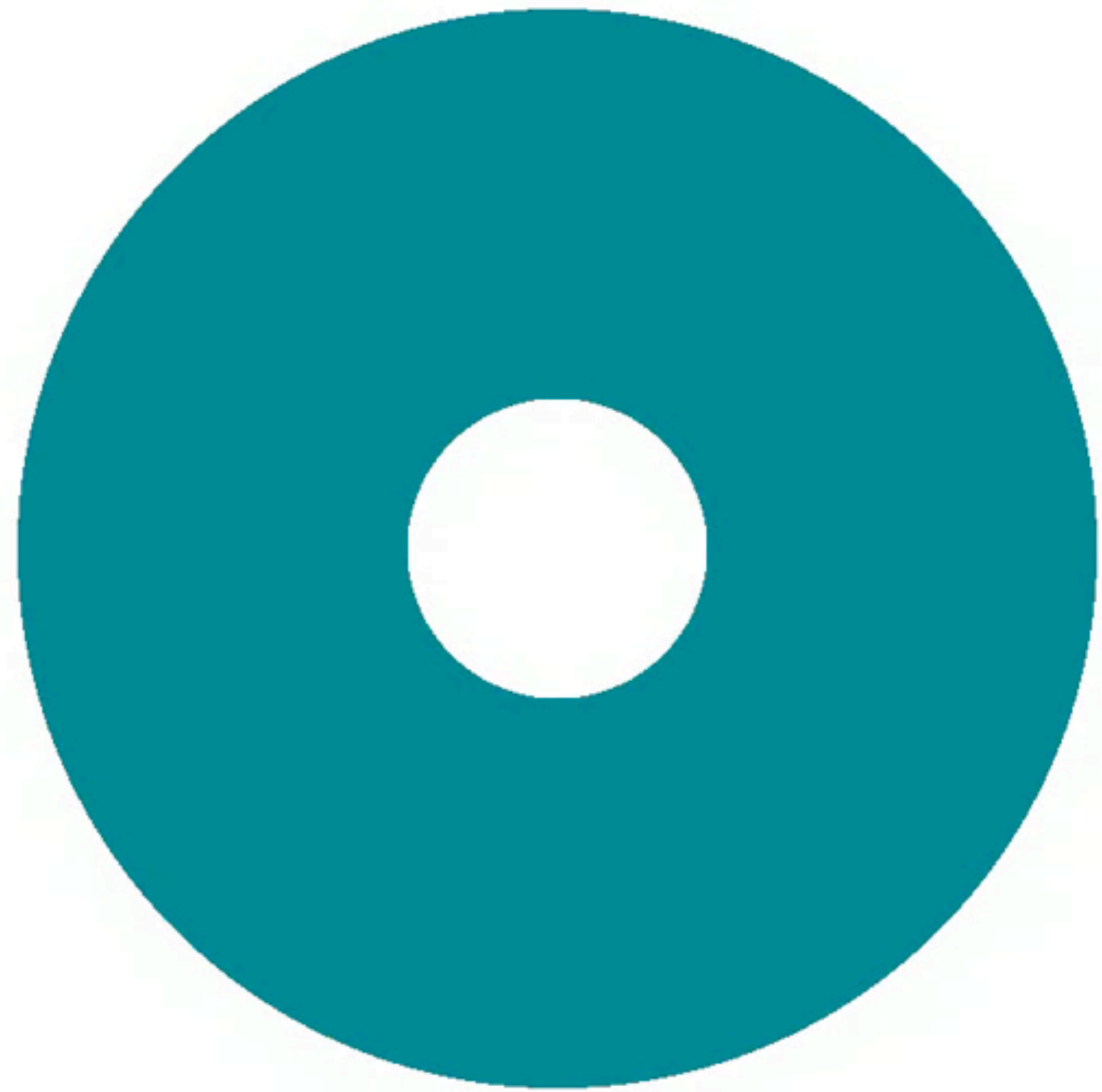
Migration - Type I

- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc



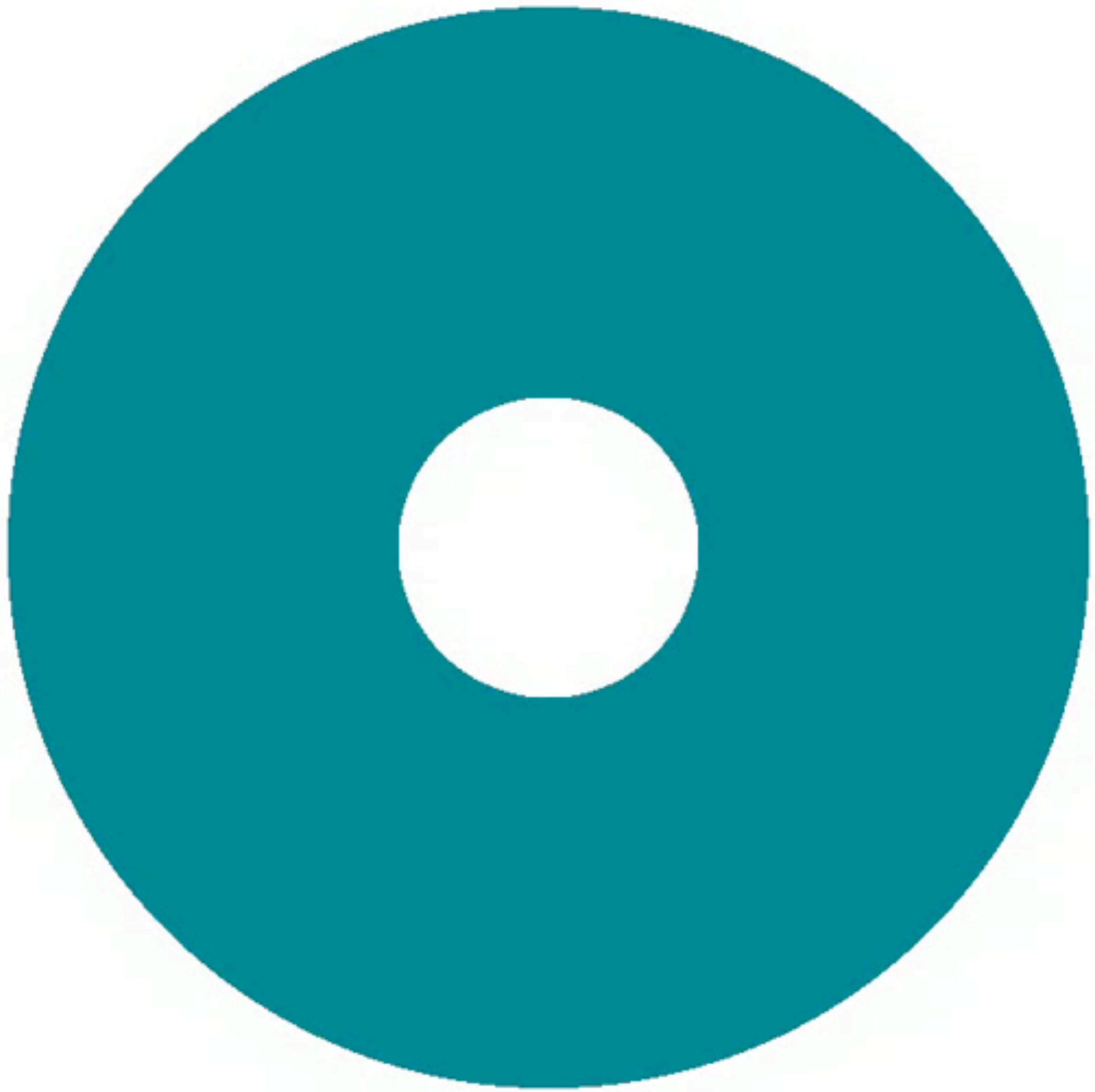
Migration - Type II

- Massive planets (typically bigger than Saturn)
- Opens a (clear) gap
- Migration rate is slow
- Follows viscous evolution of the disc



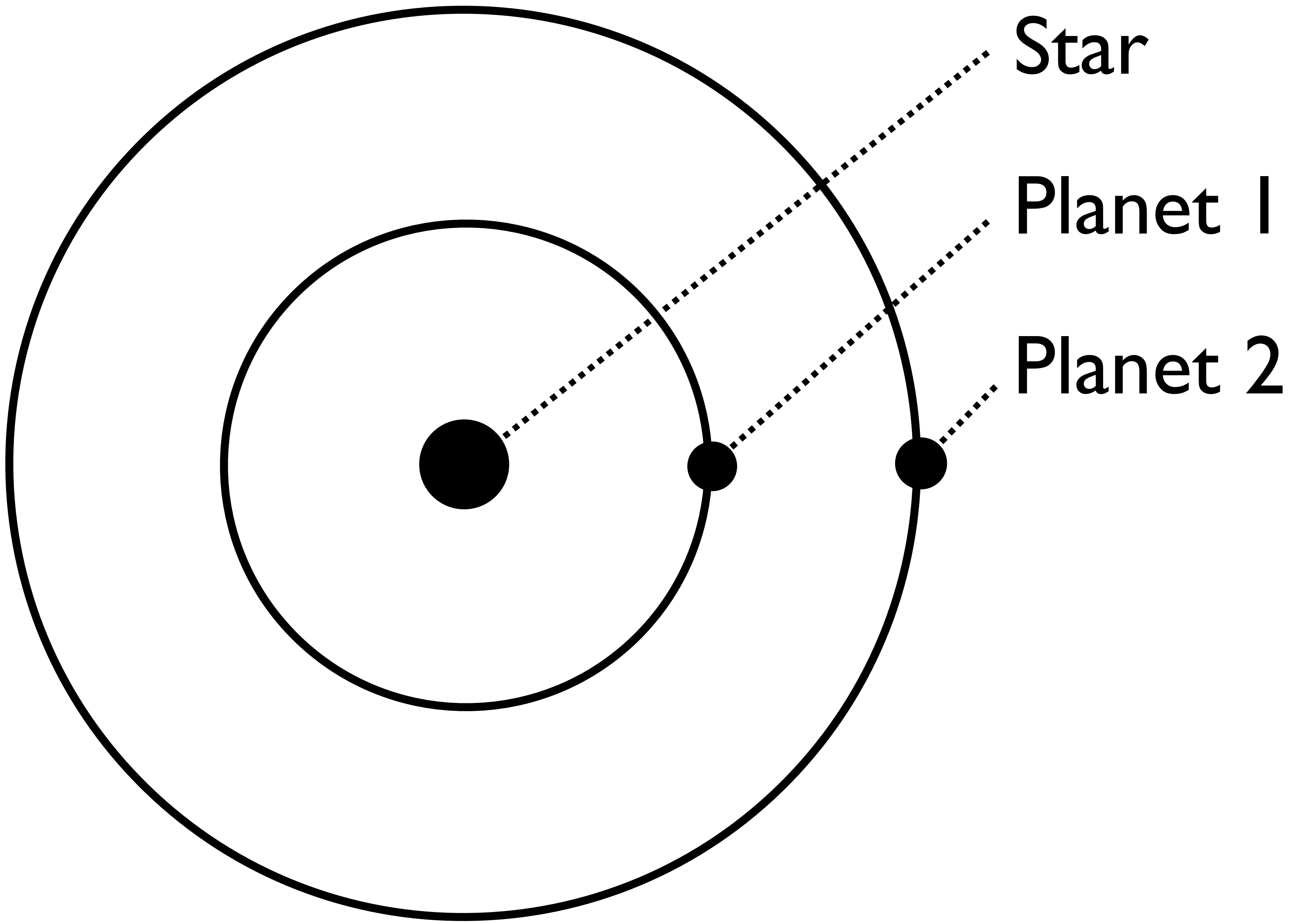
Migration - Type III

- Massive disc
- Intermediate planet mass
- Ties to open gap
- Very fast, few orbital timescales

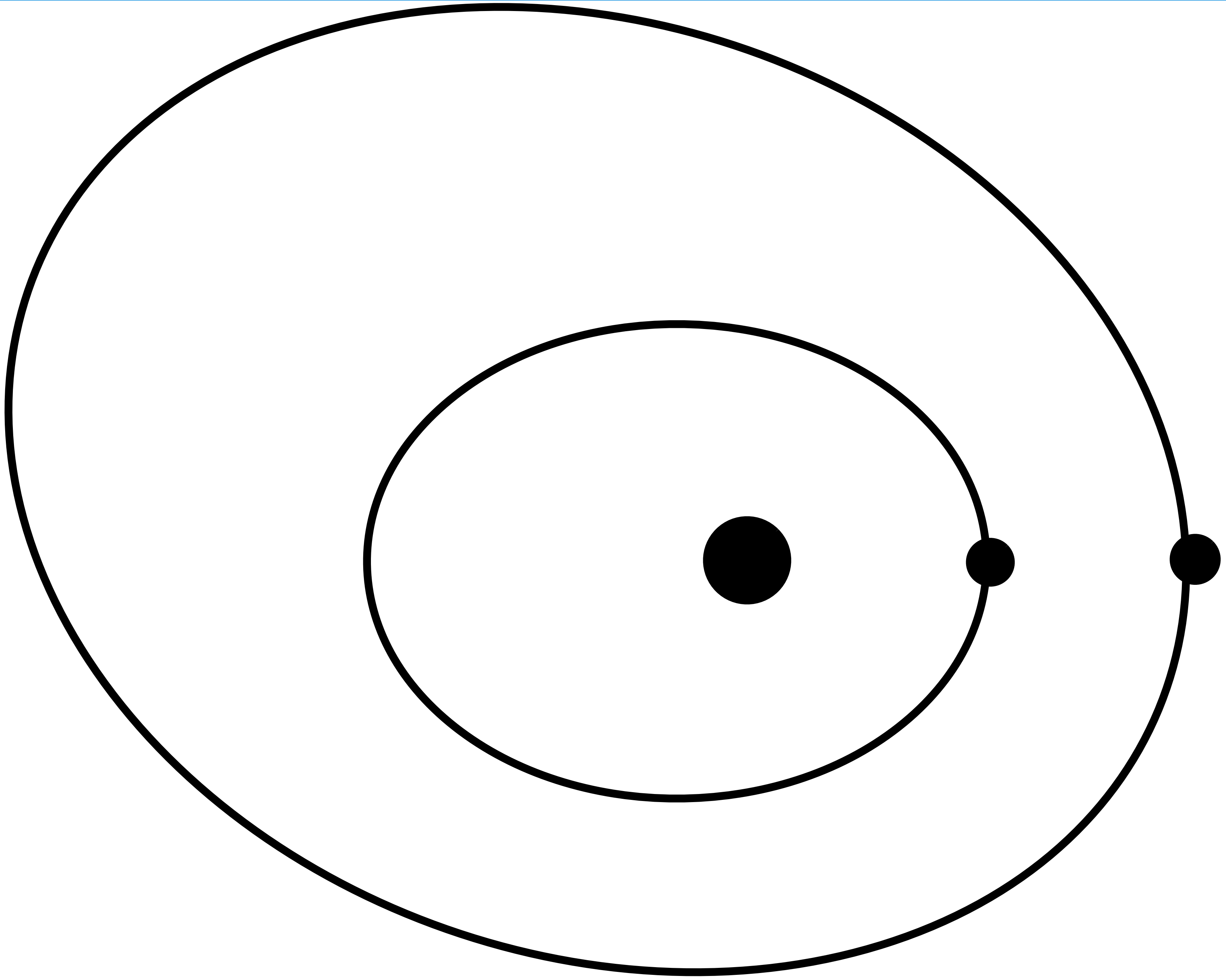


Resonance capture

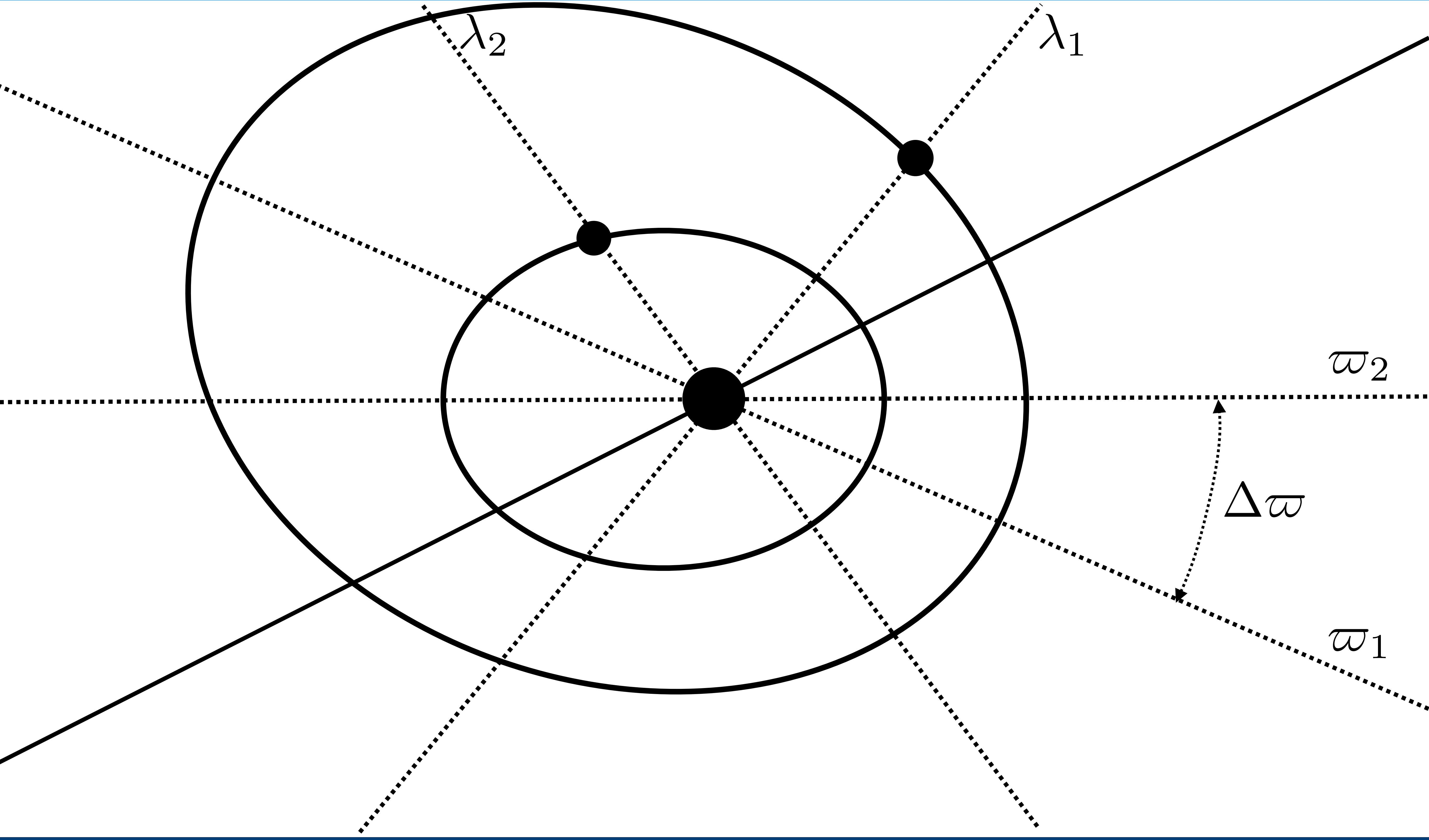
2:1 Mean Motion Resonance



2:1 Mean Motion Resonance



2:1 Mean Motion Resonance



Resonant angles

- Fast varying angles

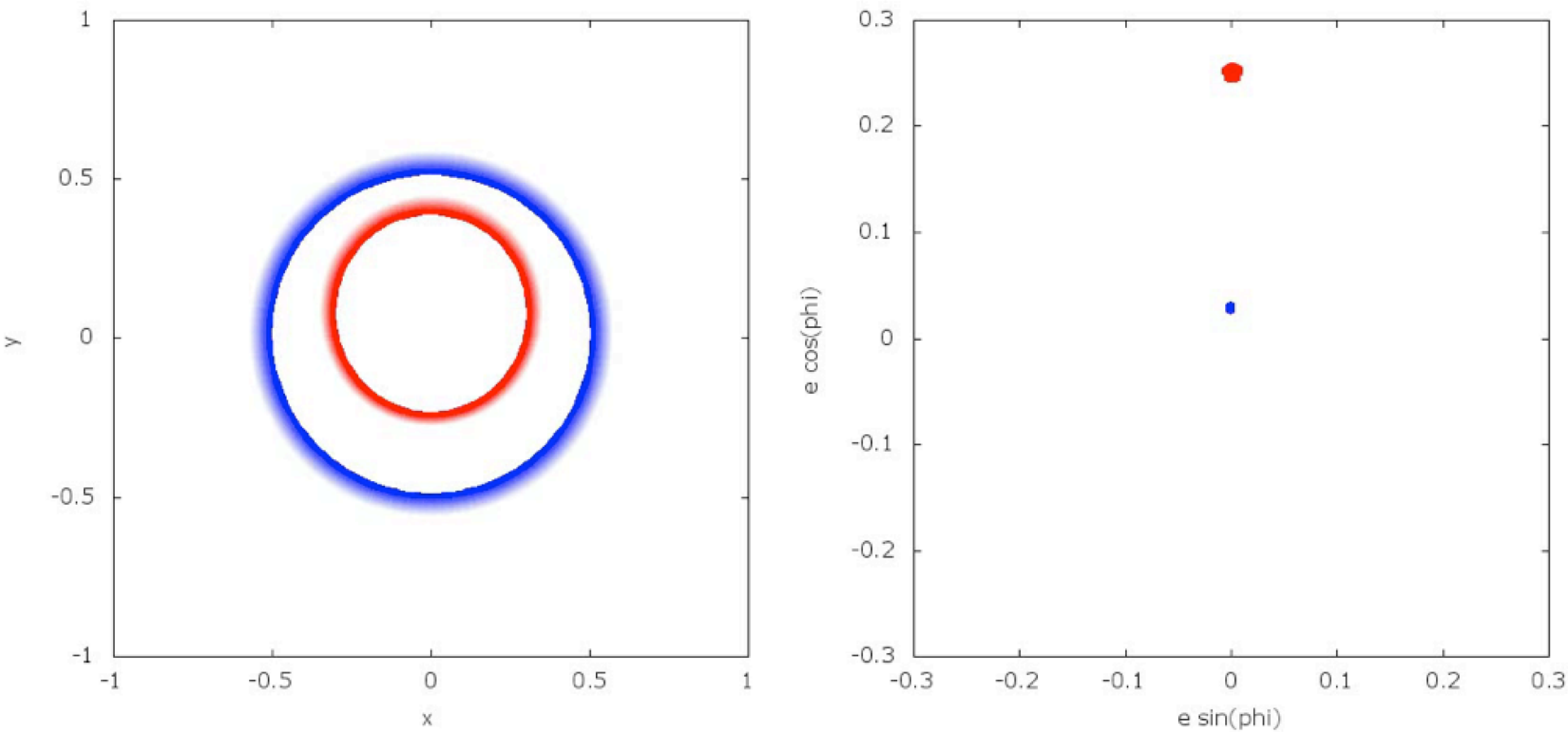
$$\lambda_1 - \varpi_1 \qquad\qquad \lambda_2 - \varpi_2$$

- Slowly varying combinations

$$\begin{aligned}
\phi_1 &= \lambda_2 - 2\lambda_1 + \varpi_2 \\
\phi_2 &= \lambda_2 - 2\lambda_1 + \varpi_1 \\
\Delta\varpi &= \varpi_1 - \varpi_2
\end{aligned}$$

- Two are linear independent

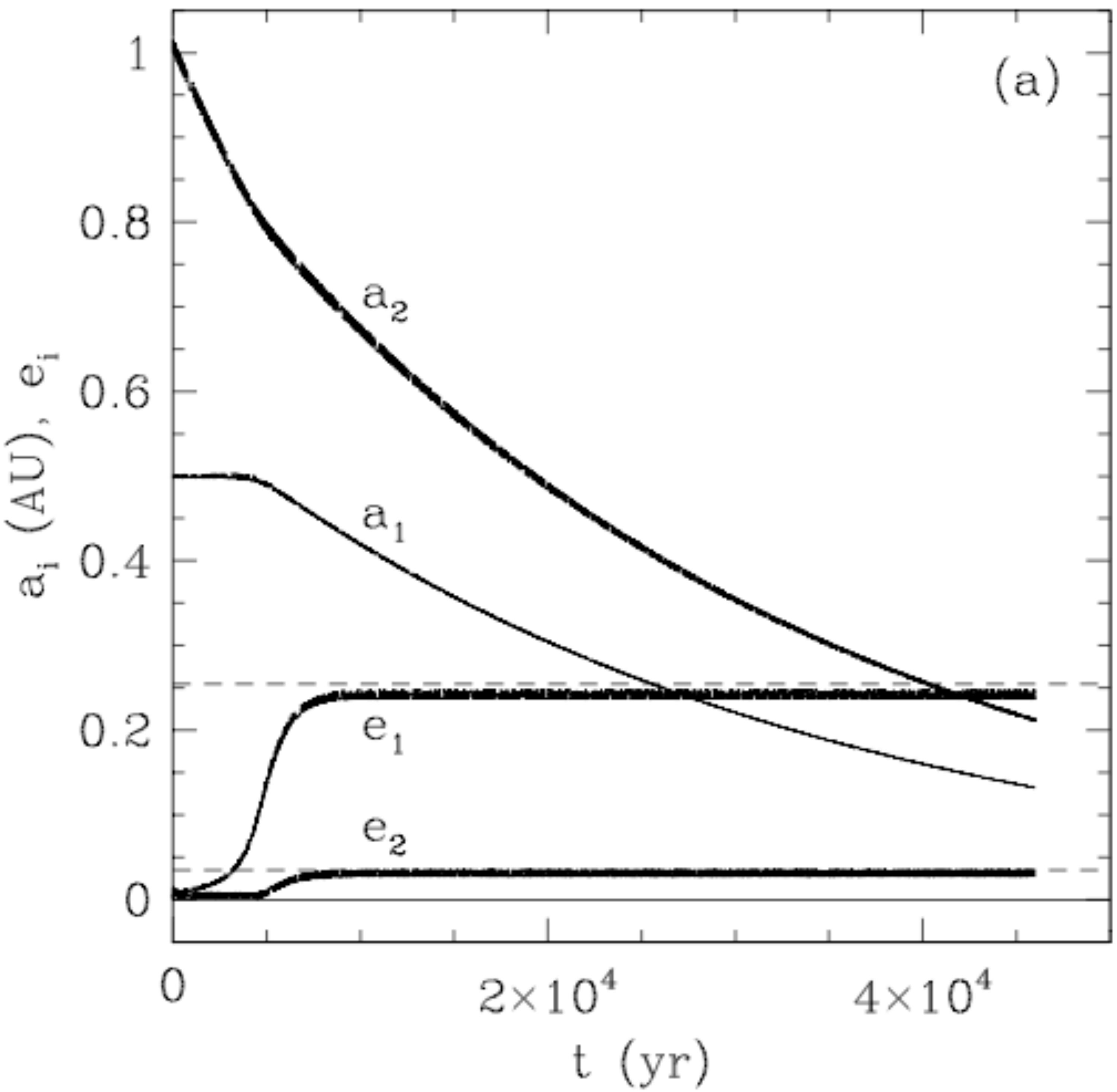
Non-turbulent resonance capture: two planets



$$\phi_1 = \lambda_2 - 2\lambda_1 + \varpi_2$$

parameters of GJ 876

GJ 876



Lee & Peale 2002

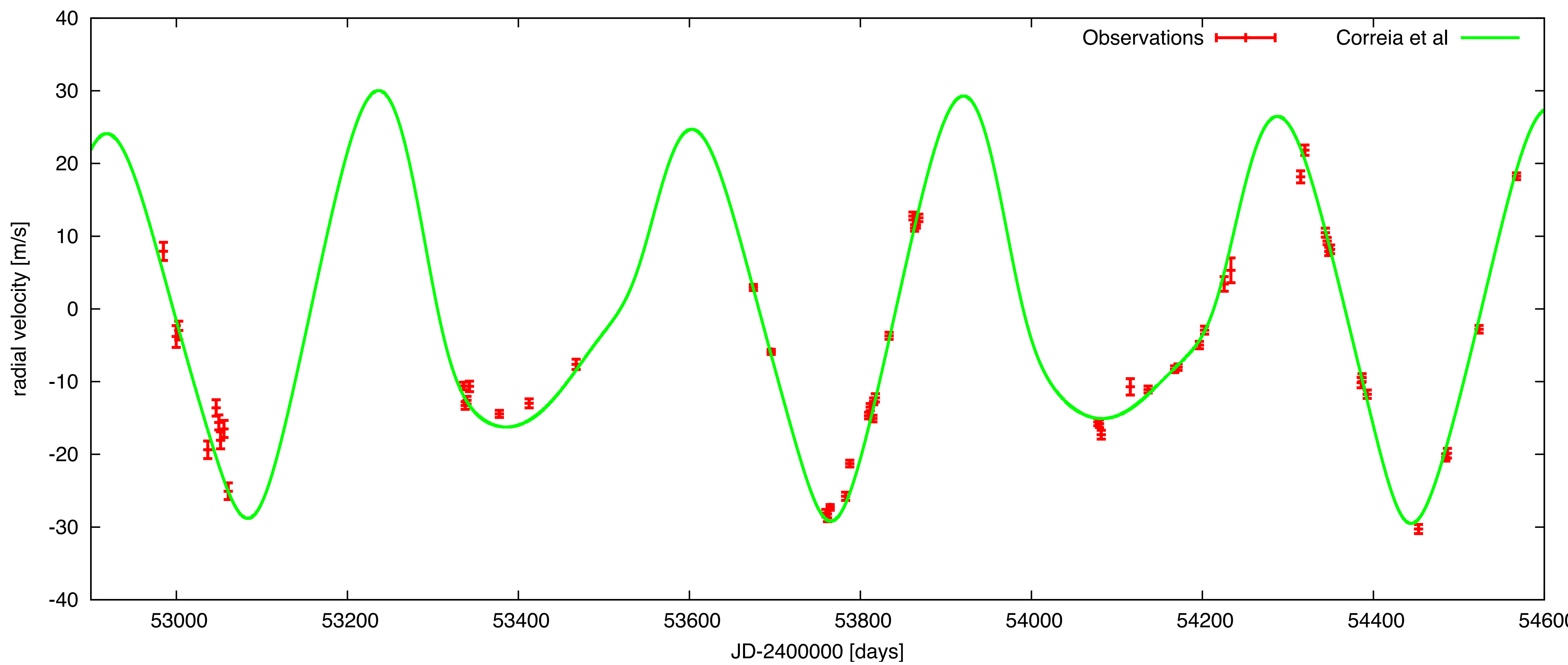
Take home message I

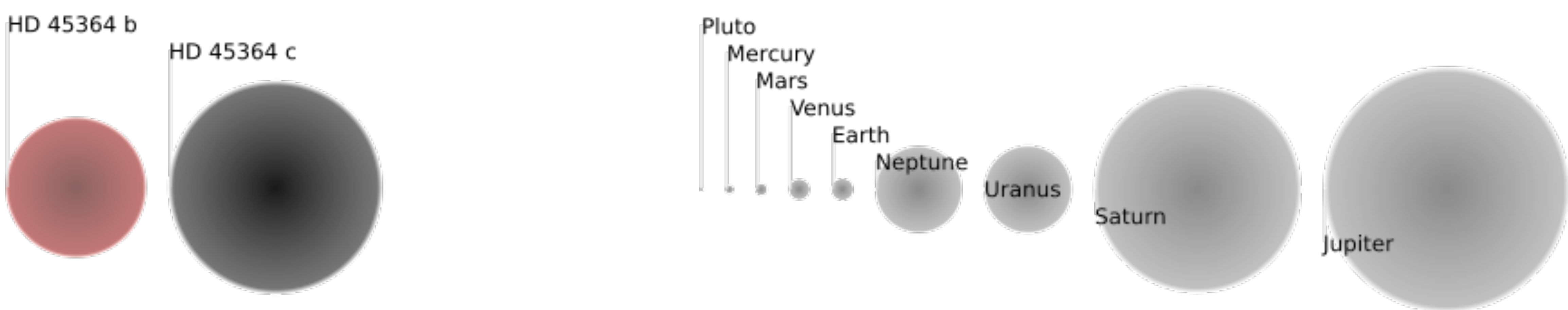
planet + disc = migration

2 planets + migration = resonance

HD 45364

HD45364





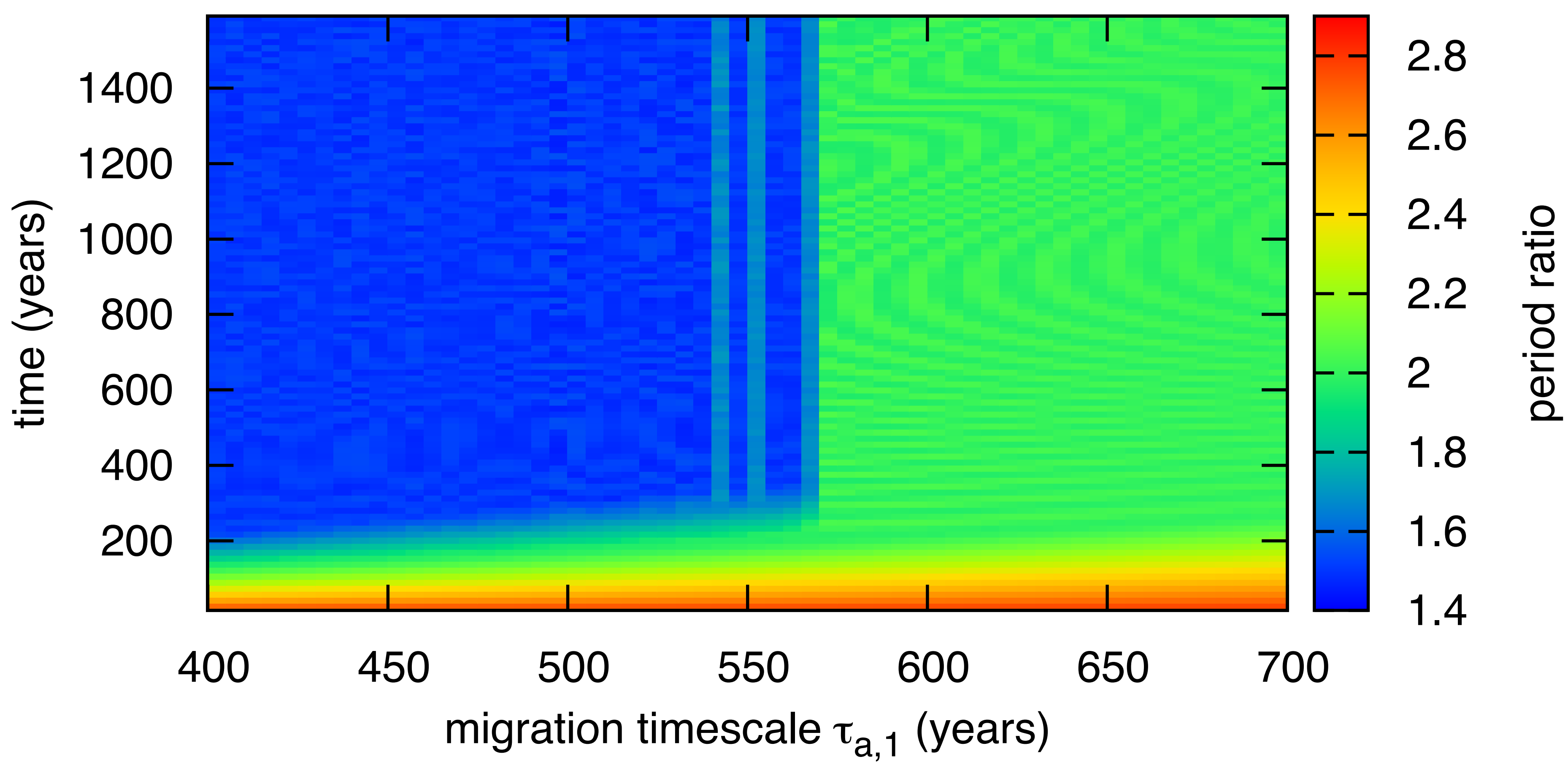
Correia et al 2009, Visual Exoplanet Catalogue

Formation scenario for HD45364

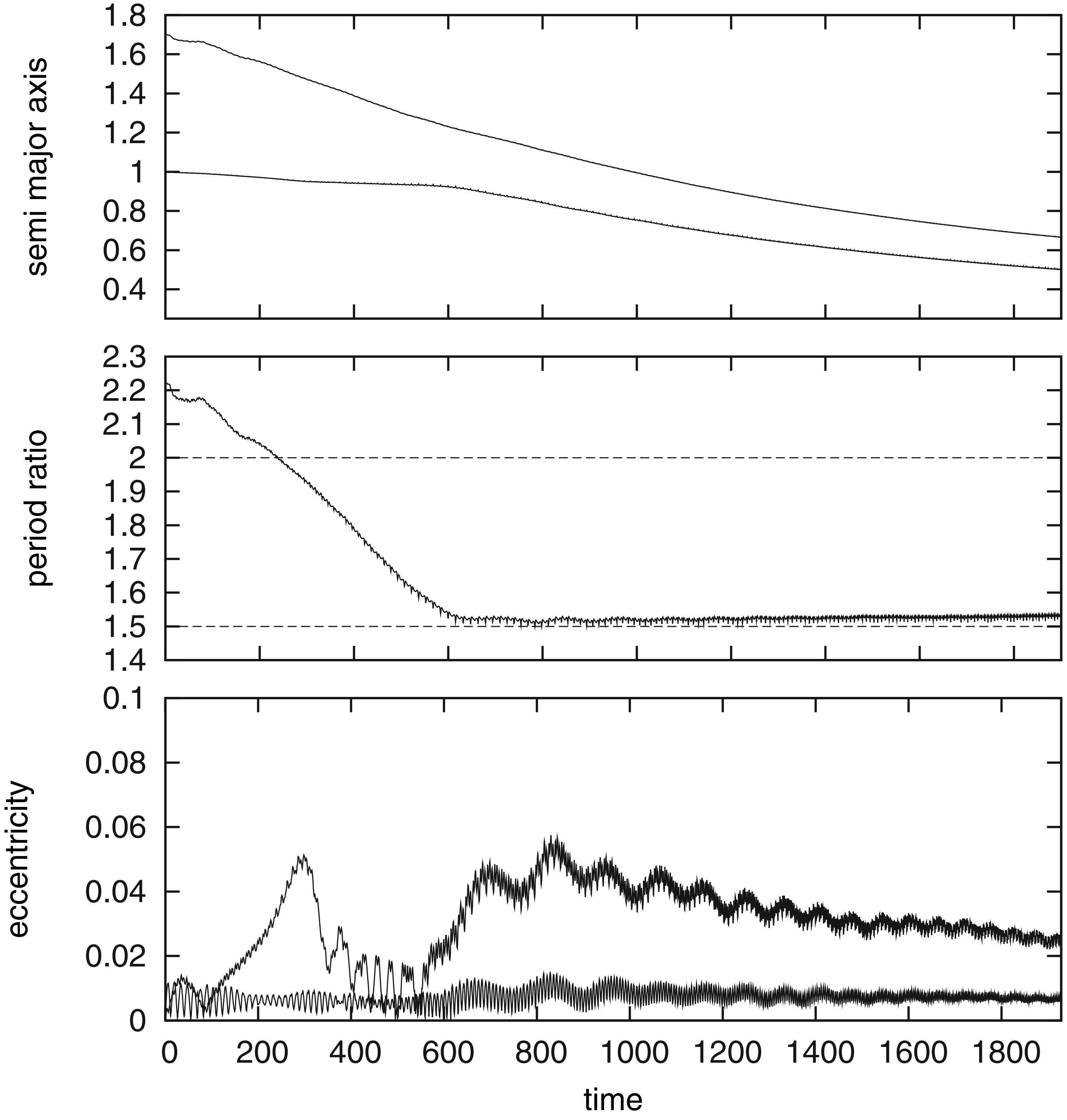
- Two migrating planets
- Infinite number of resonances

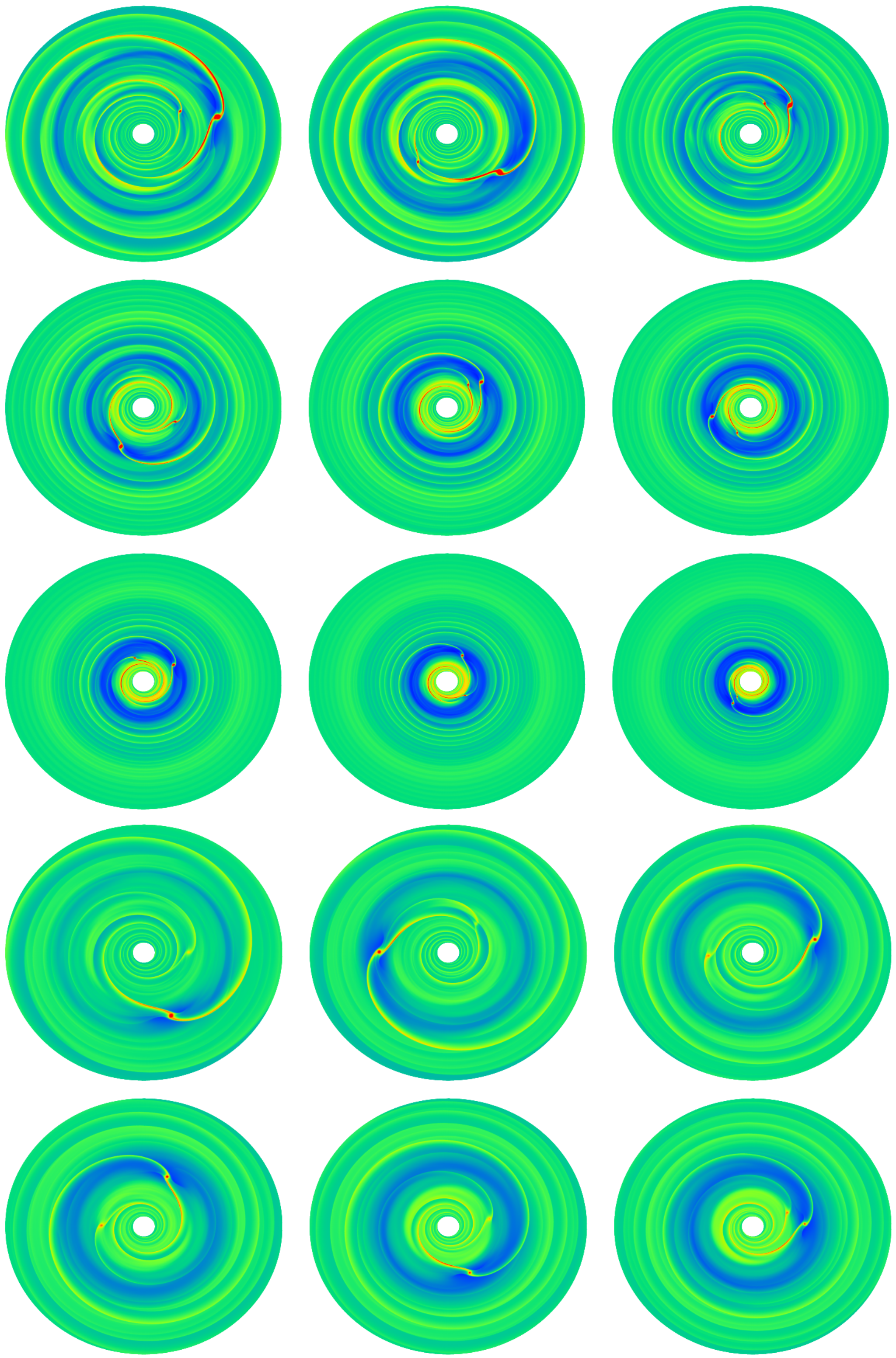
1:2 7:8 3:2 1:3 3:4

- Migration speed is crucial
- Resonance width and libration period define critical migration rate



Formation scenario for HD45364





Rein, Papaloizou & Kley 2010

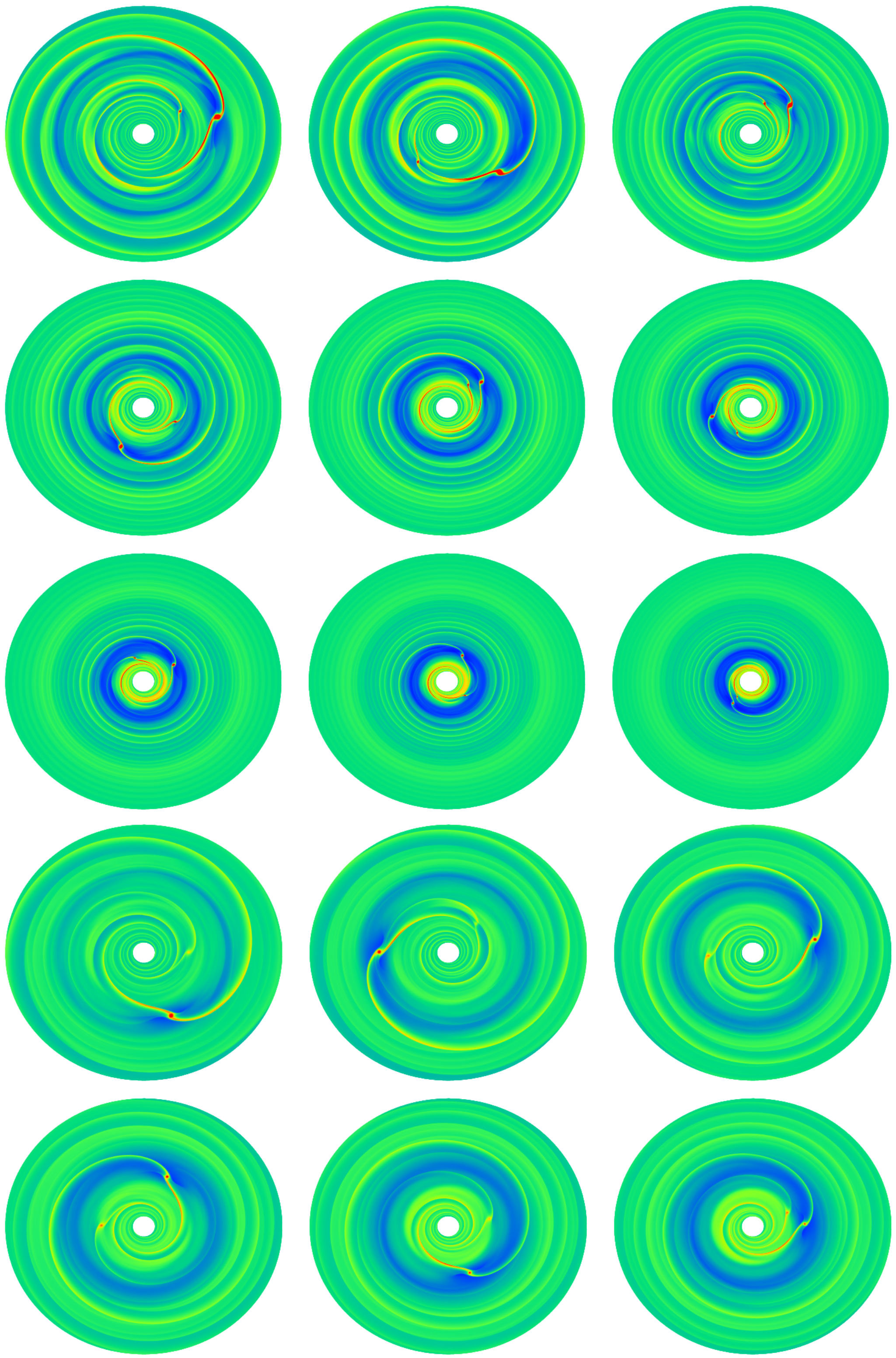
Formation scenario for HD45364

Massive disc (5 times MMSN)

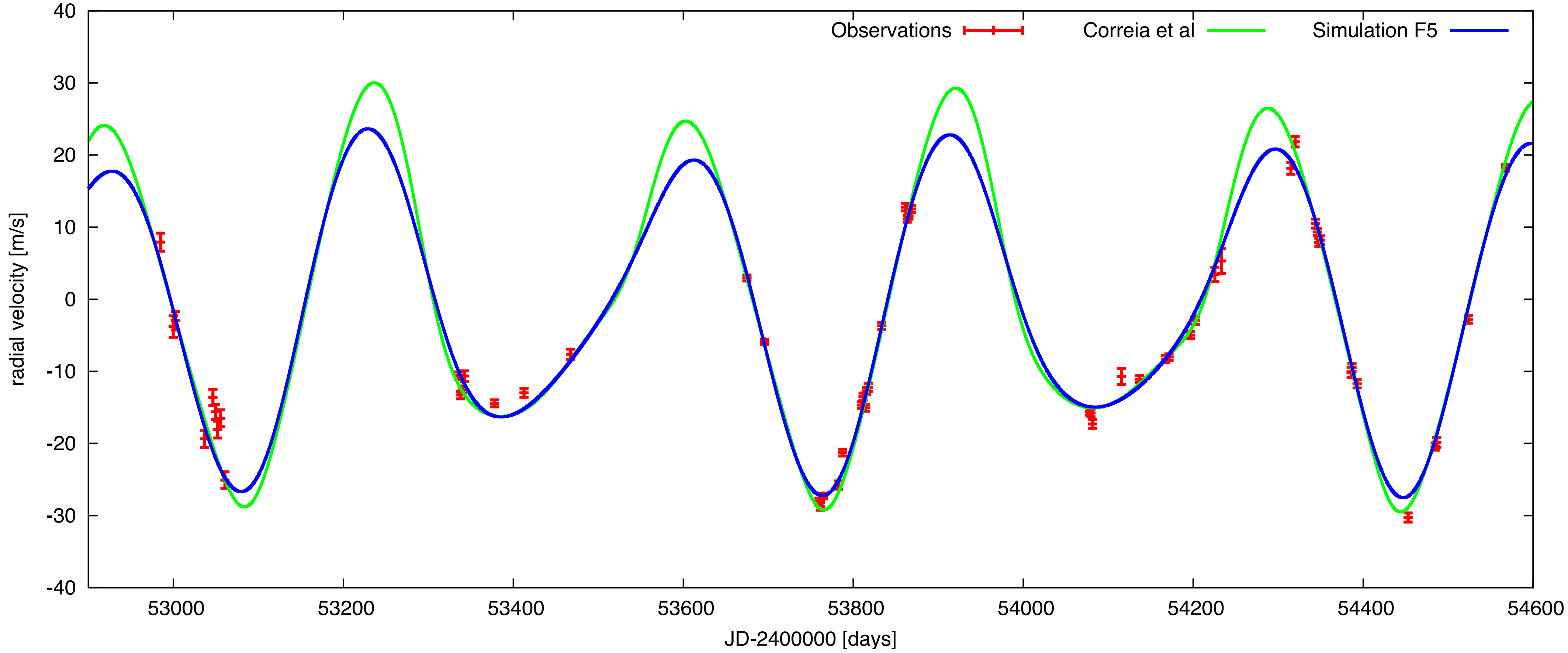
- Short, rapid Type III migration
- Passage of 2:1 resonance
- Capture into 3:2 resonance

Large scale-height (0.07)

- Slow Type I migration once in resonance
- Resonance is stable
- Consistent with radiation hydrodynamics



Formation scenario leads to a better 'fit'



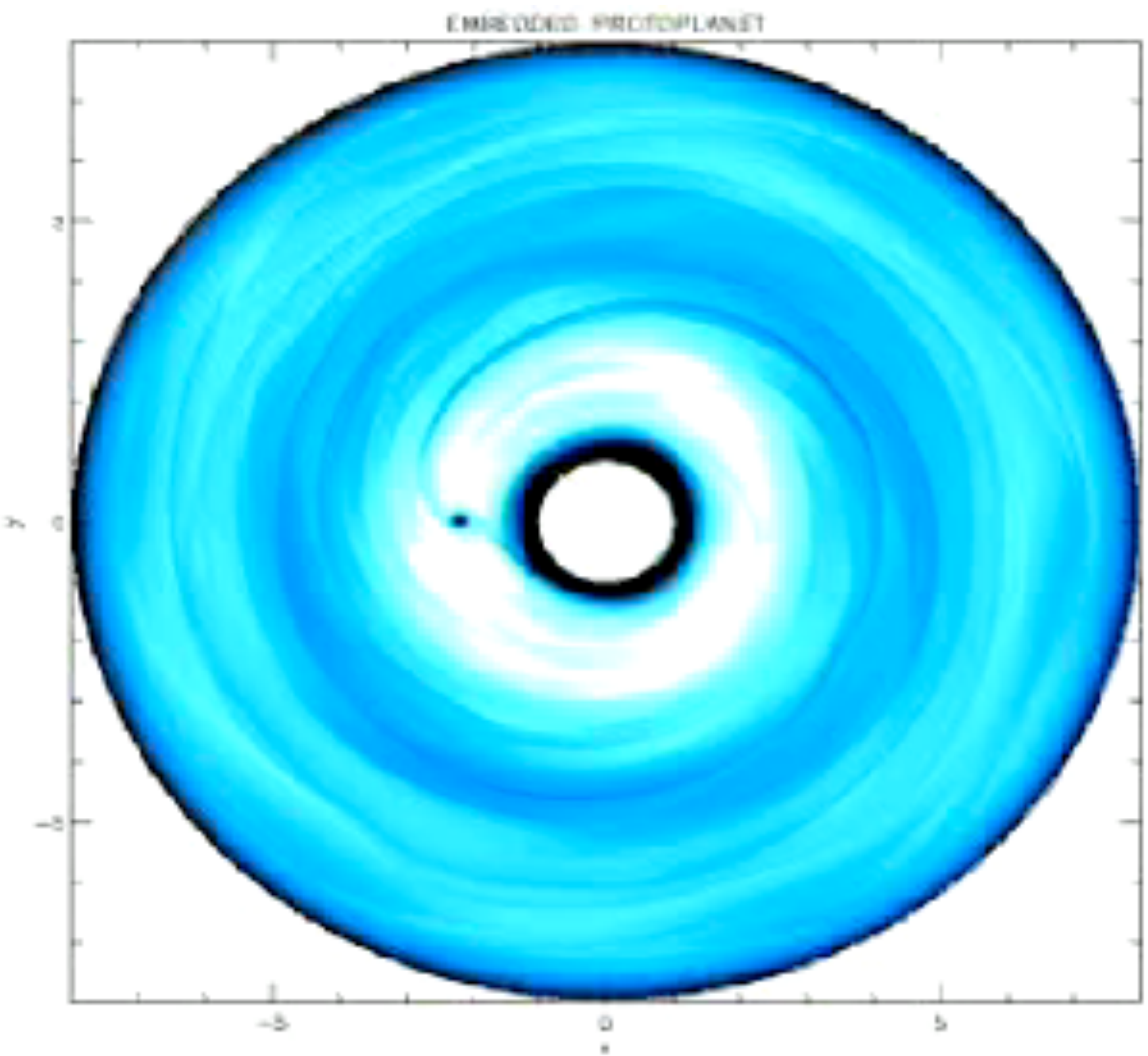
Parameter	Unit	Correia et al. (2009) b	c	Simulation **F5** b	c
$M \sin i$	$[\mathrm{M_{Jup}}]$	0.1872	0.6579	0.1872	0.6579
M_*	$[M_\odot]$	0.82		0.82	
a	[AU]	0.6813	0.8972	0.6804	0.8994
e		0.17 ± 0.02	0.097 ± 0.012	0.036	0.017
λ	[deg]	105.8 ± 1.4	269.5 ± 0.6	352.5	153.9
ϖ^a	[deg]	162.6 ± 6.3	7.4 ± 4.3	87.9	292.2
$\sqrt{\chi^2}$		2.79		2.76^b (3.51)	
Date	[JD]	2453500		2453500	

Rein, Papaloizou & Kley 2010

Migration in a turbulent disc

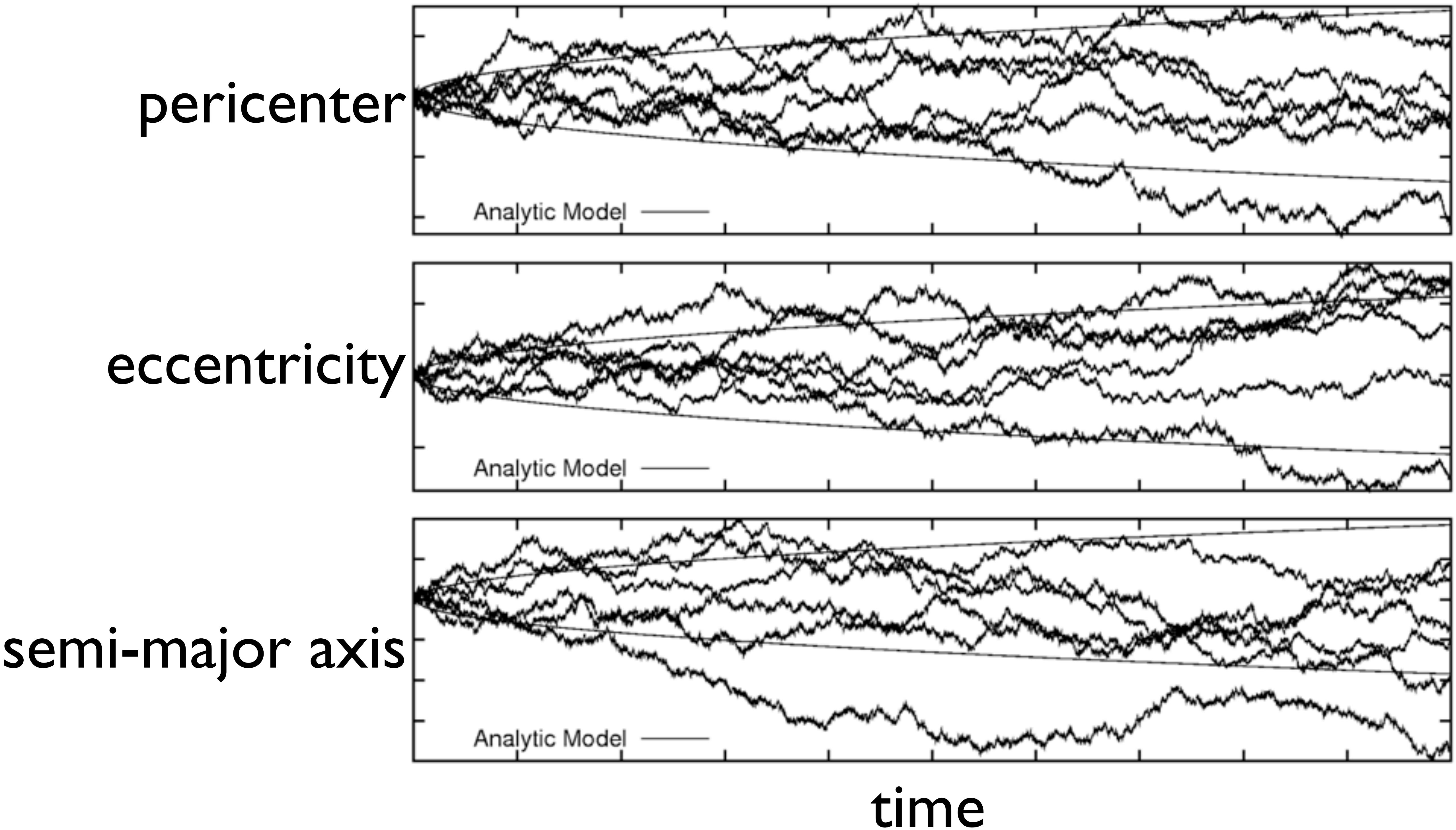
Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces



Animation from Nelson & Papaloizou 2004
Random forces measured by Laughlin et al. 2004, Nelson 2005, Oischi et al. 2007

Random walk



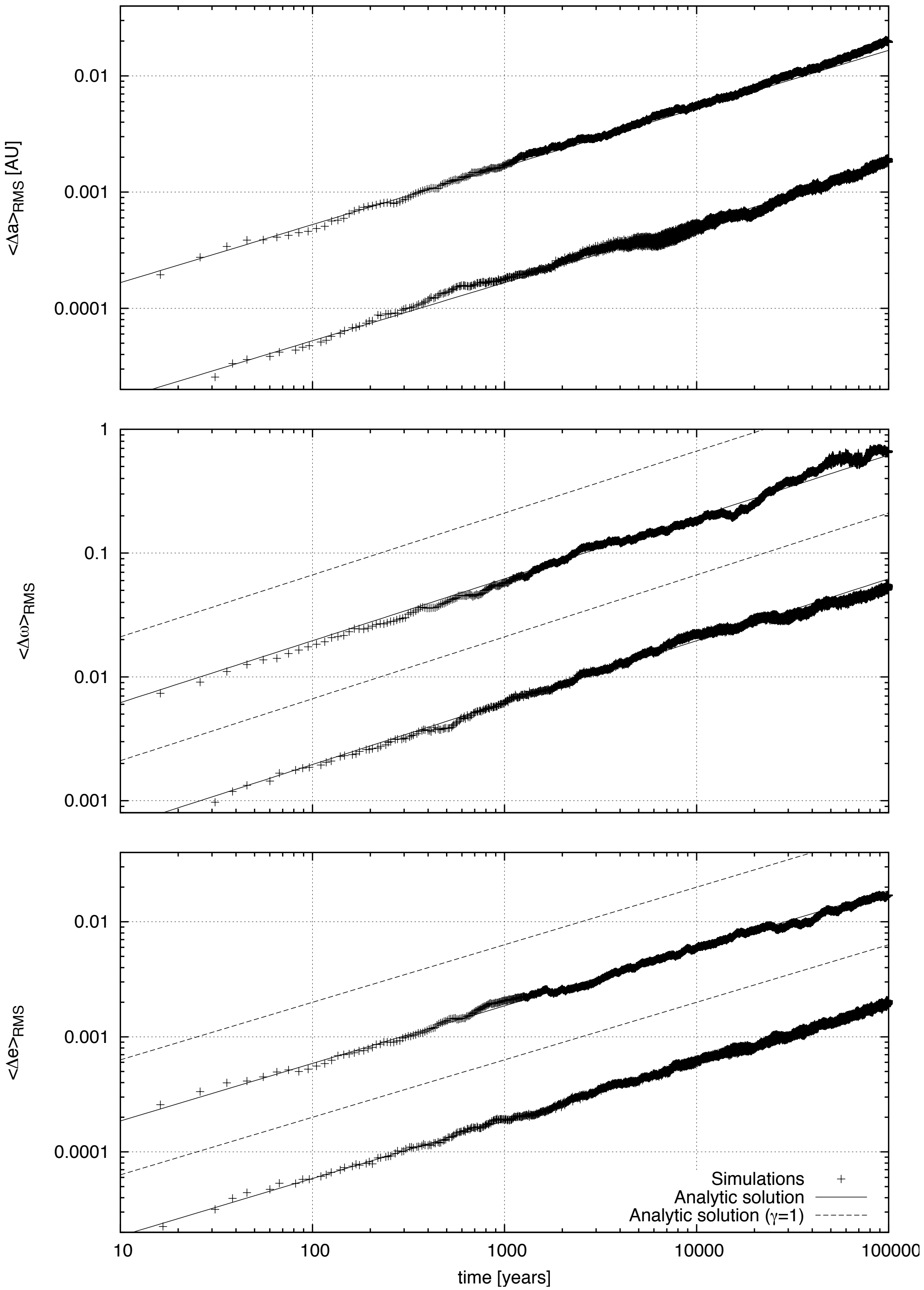
Rein & Papaloizou 2009

Correction factors are important

$$(\Delta a)^2 = 4\frac{Dt}{n^2}$$

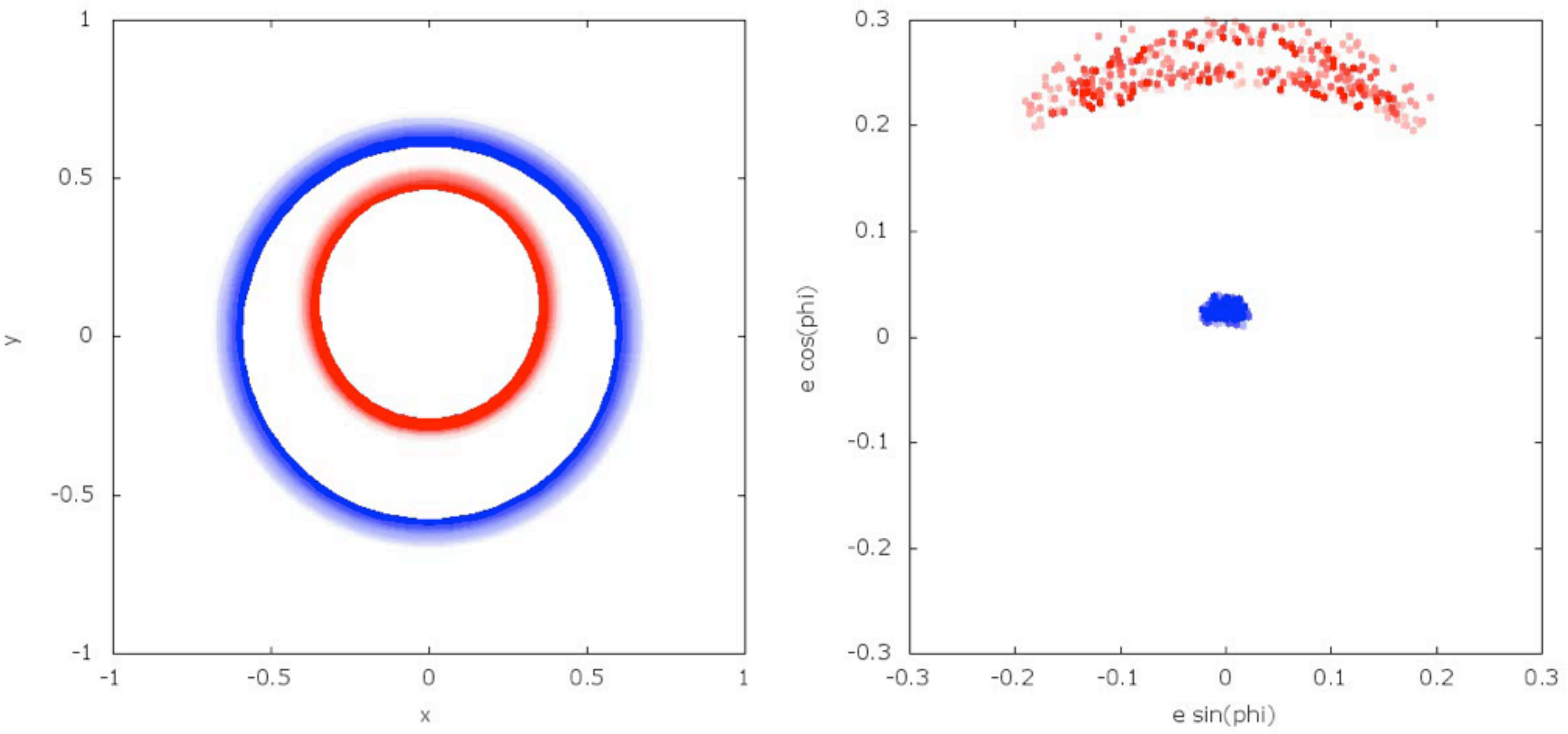
$$(\Delta \varpi)^2 = \frac{2.5}{e^2}\frac{\gamma Dt}{n^2 a^2}$$

$$(\Delta e)^2 = 2.5\frac{\gamma Dt}{n^2 a^2}$$

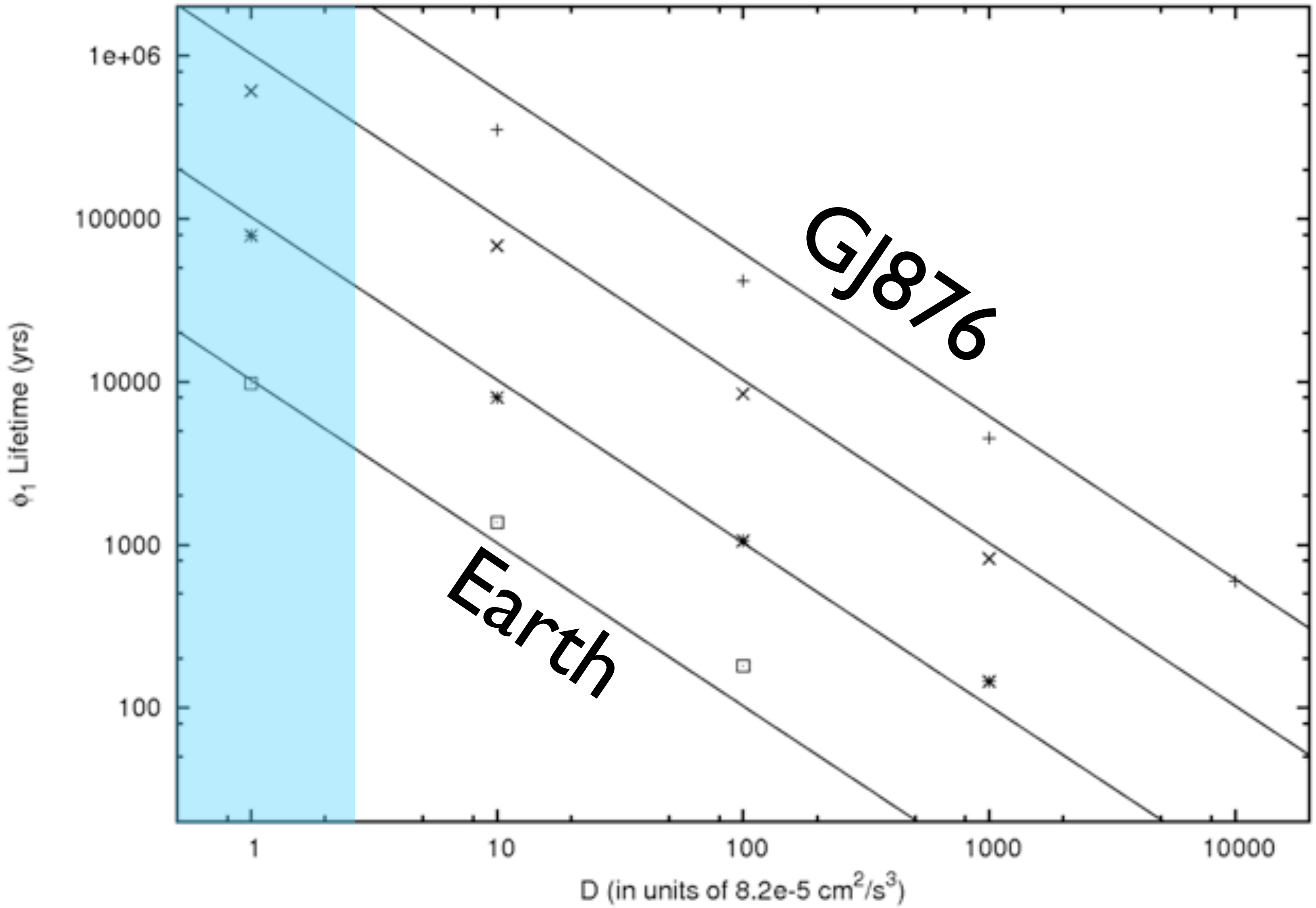


Rein & Papaloizou 2009, Adams et al 2009, Rein 2010

Two planets: turbulent resonance capture



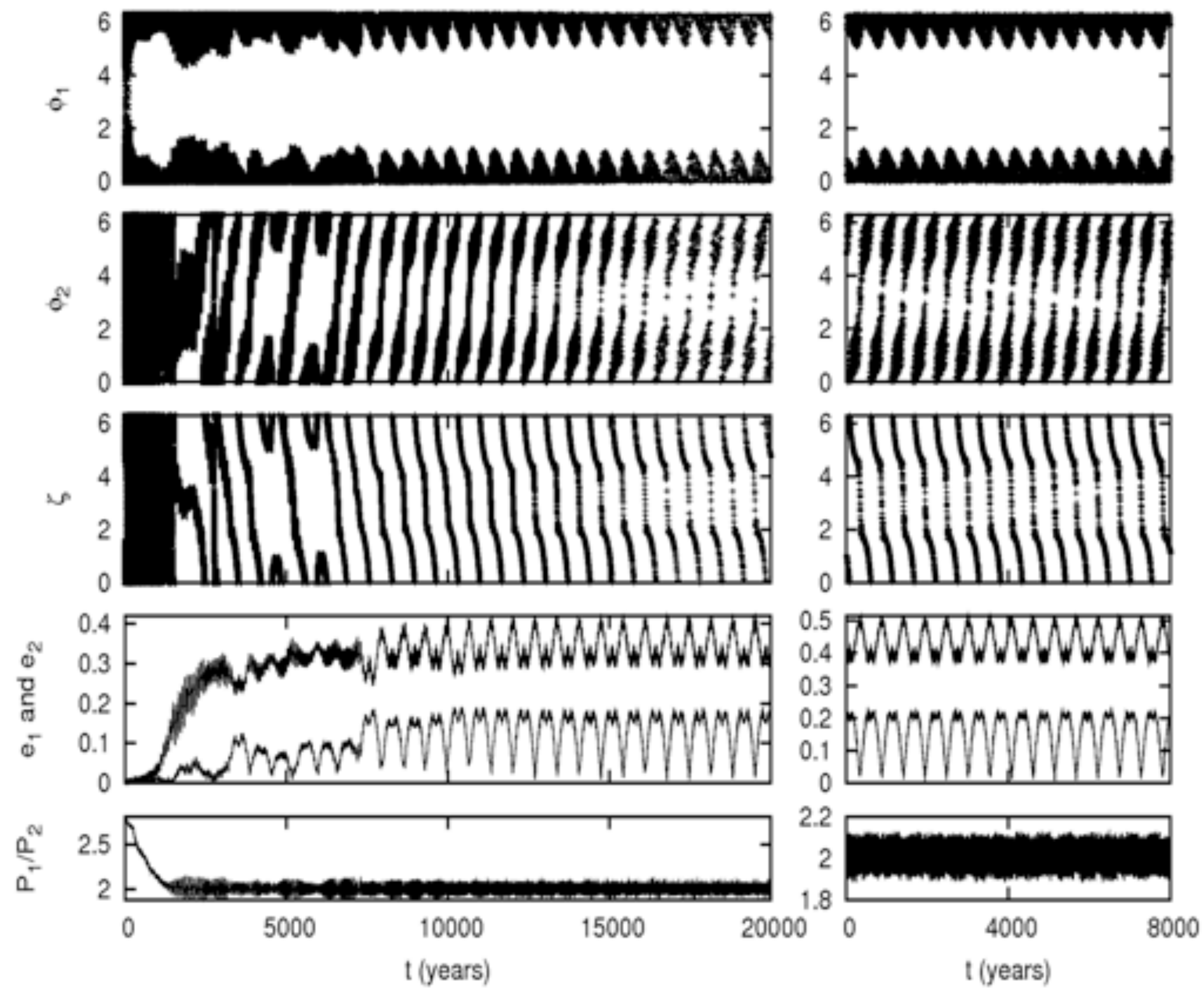
Multi-planetary systems in mean motion resonance



- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

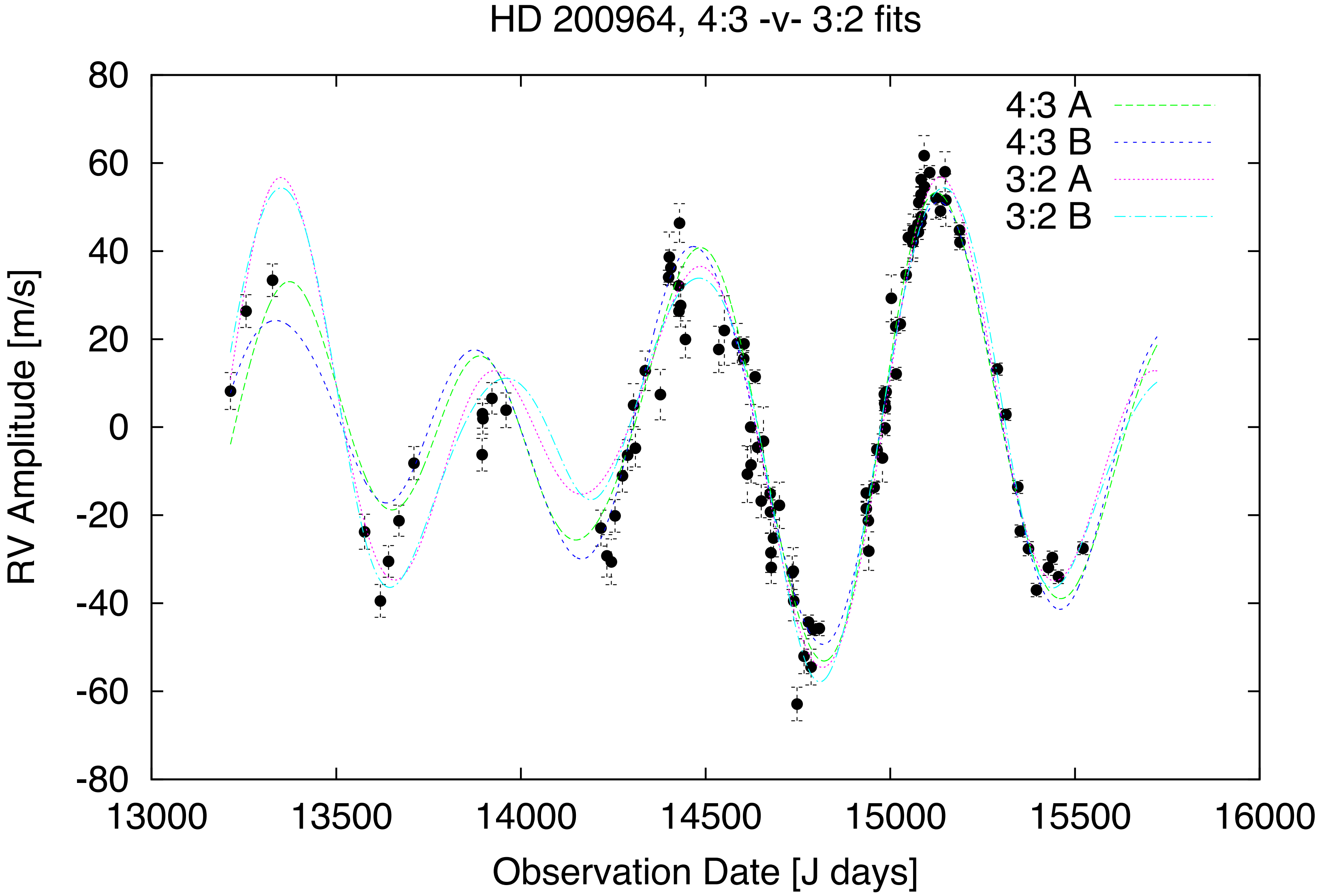
Modification of libration patterns

- HD128311 has a very peculiar libration pattern
- Can not be reproduced by convergent migration alone
- Turbulence can explain it
- More multi-planetary systems needed for statistical argument



HD200964
The impossible system?

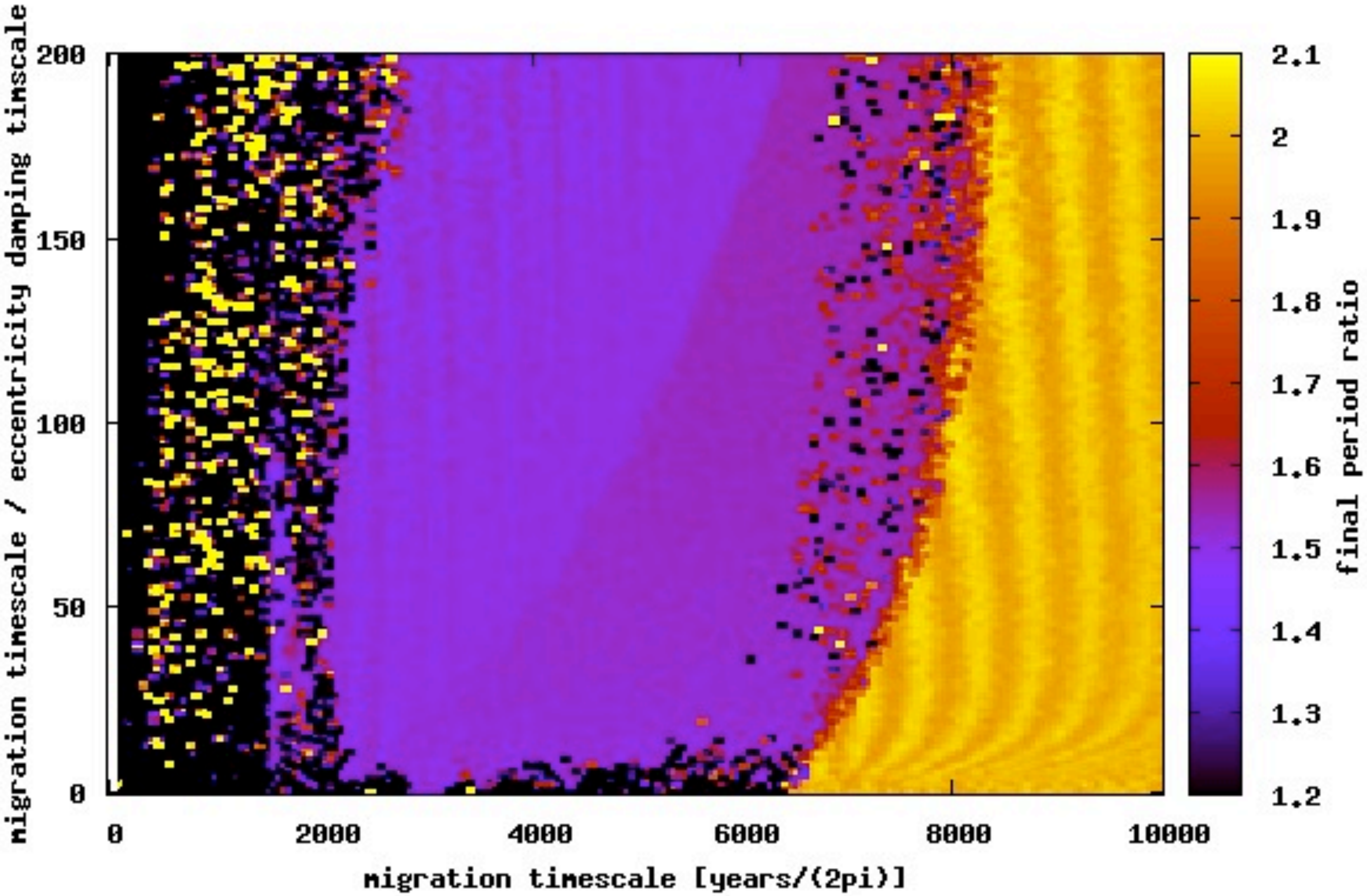
Radial velocity curve of HD200964



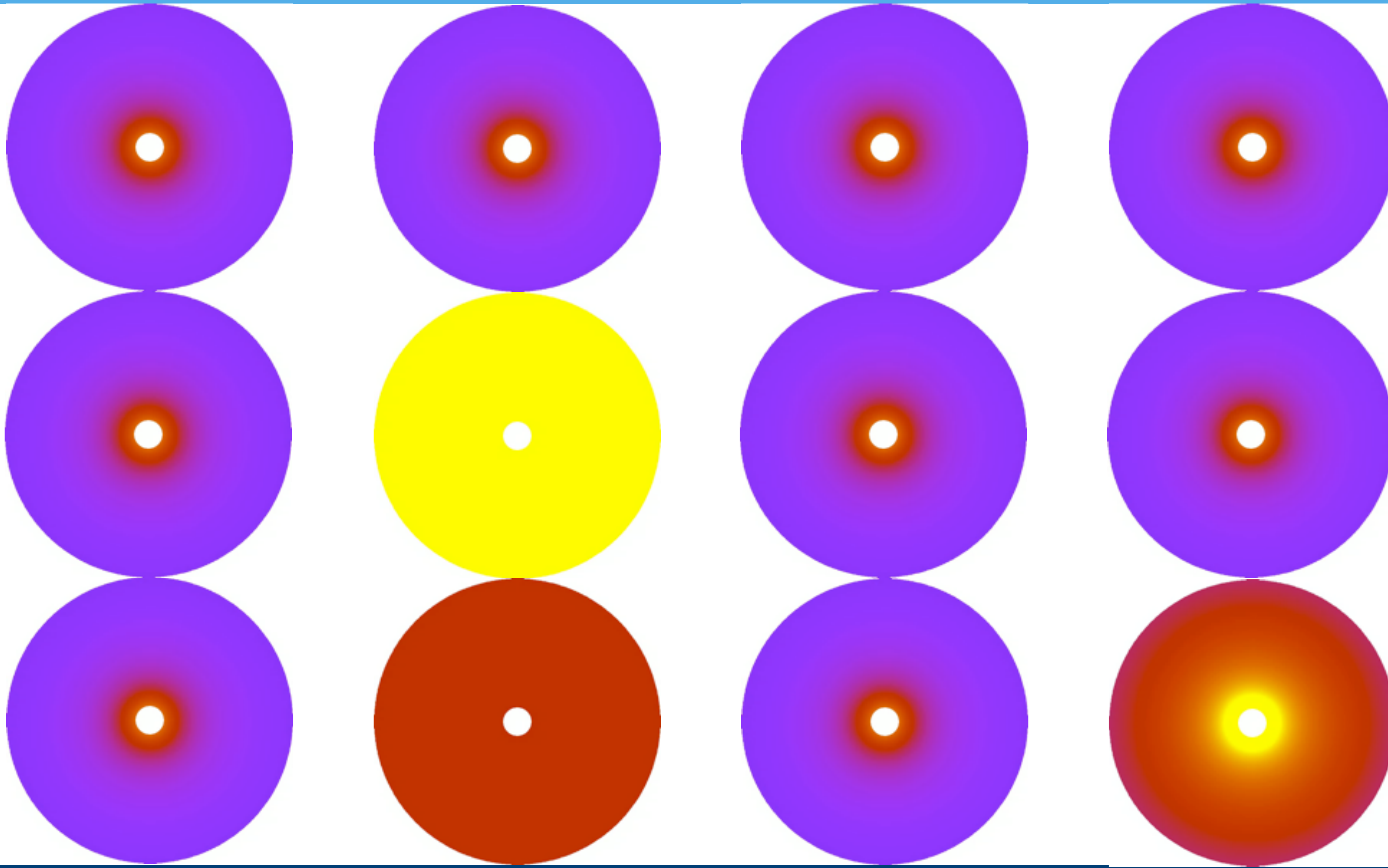
- Two massive planets 1.8 M_{Jup} and 0.9 M_{Jup}
- Period ratio either 3:2 or 4:3
- Another similar system, to be announced soon
- How common is 4:3?
- Formation?

Plot by Matthew Payne

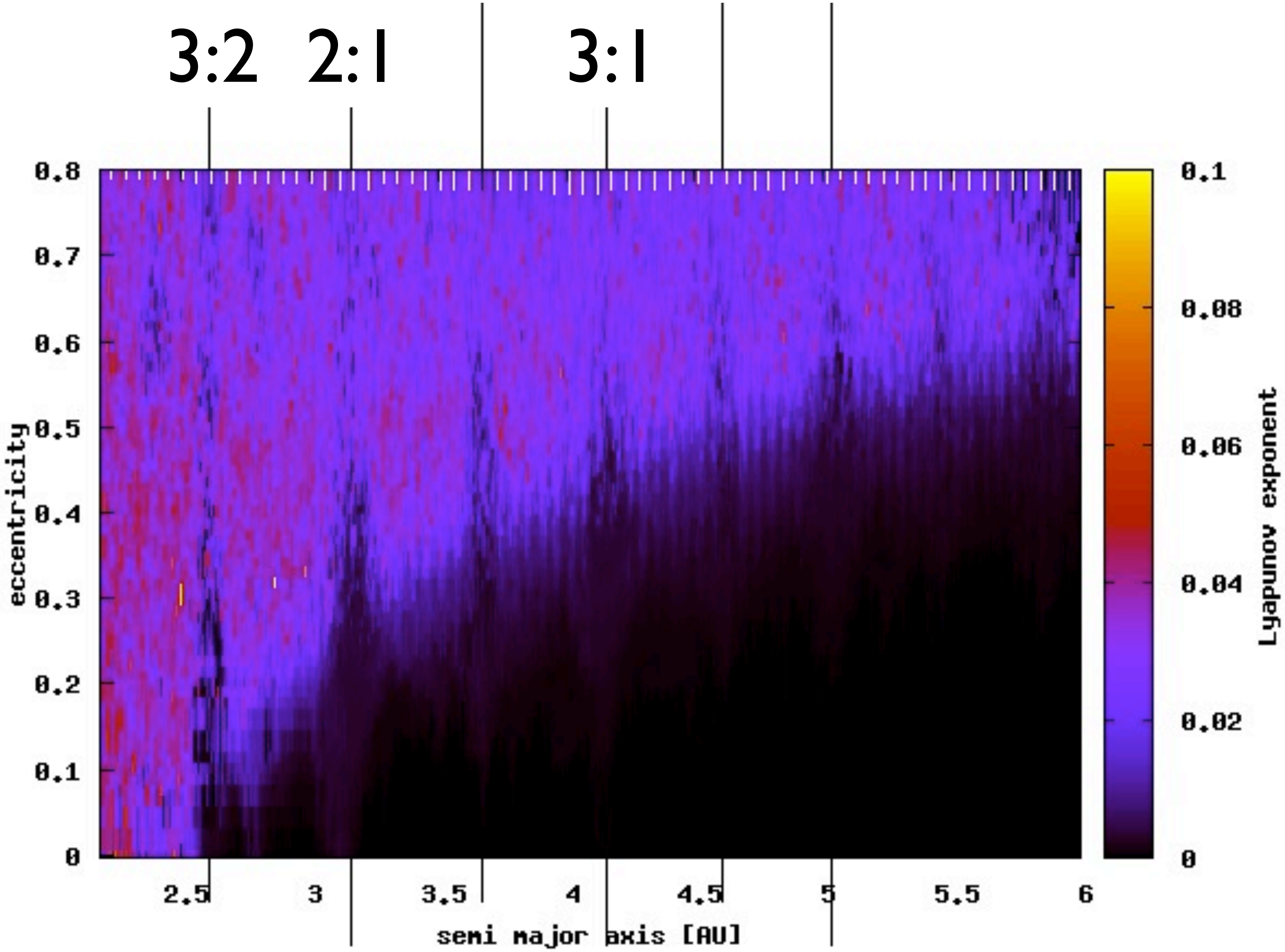
Standard disc migration doesn't work



Hydrodynamical simulations



Stability of HD200964



HD200964

- In situ formation?
- Main accretion while in 4:3 resonance?
- Planet planet scattering?
- A third planet?
- Observers screwed up?

Take home message II

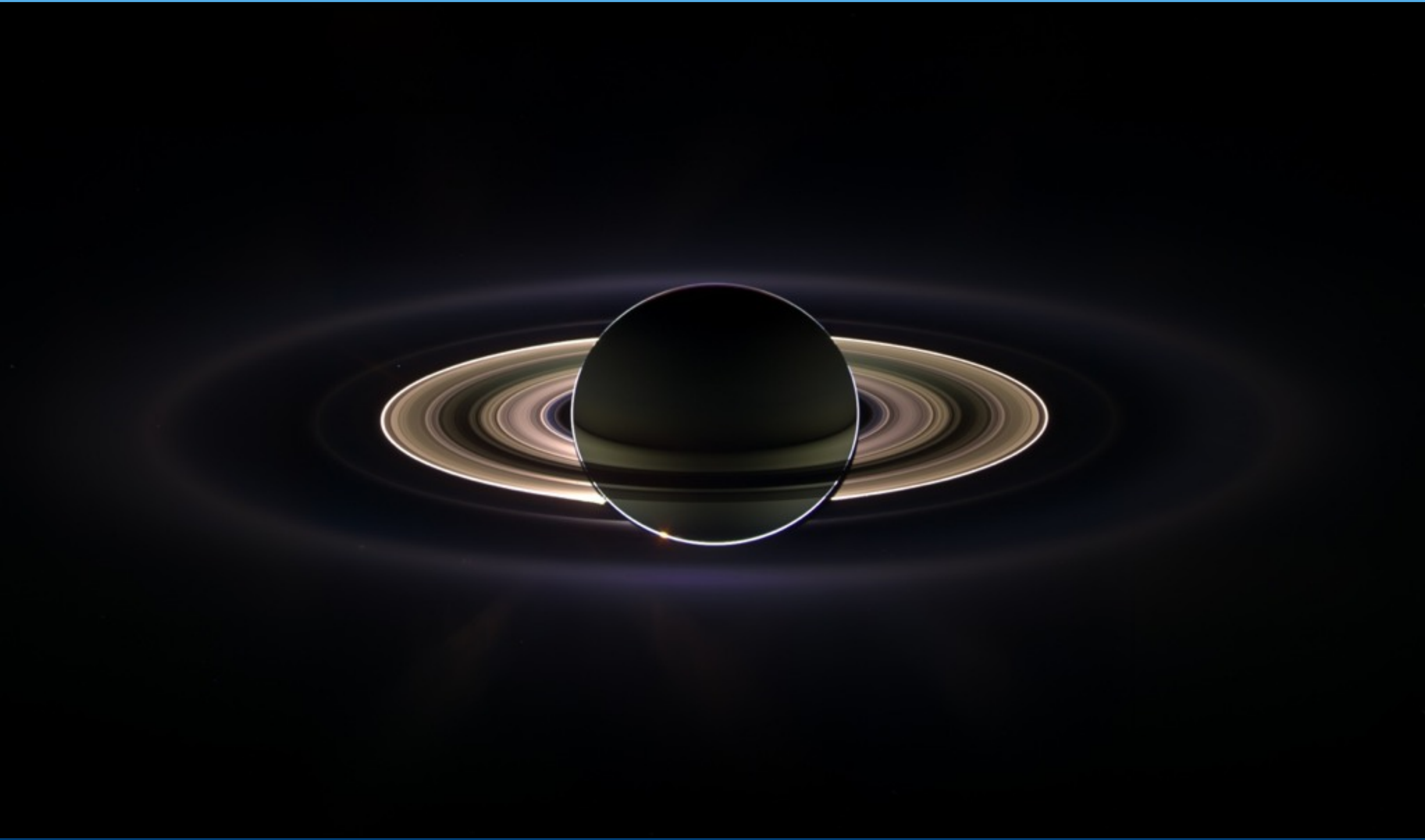
dynamical state of planetary systems

$\longleftrightarrow$

formation scenario

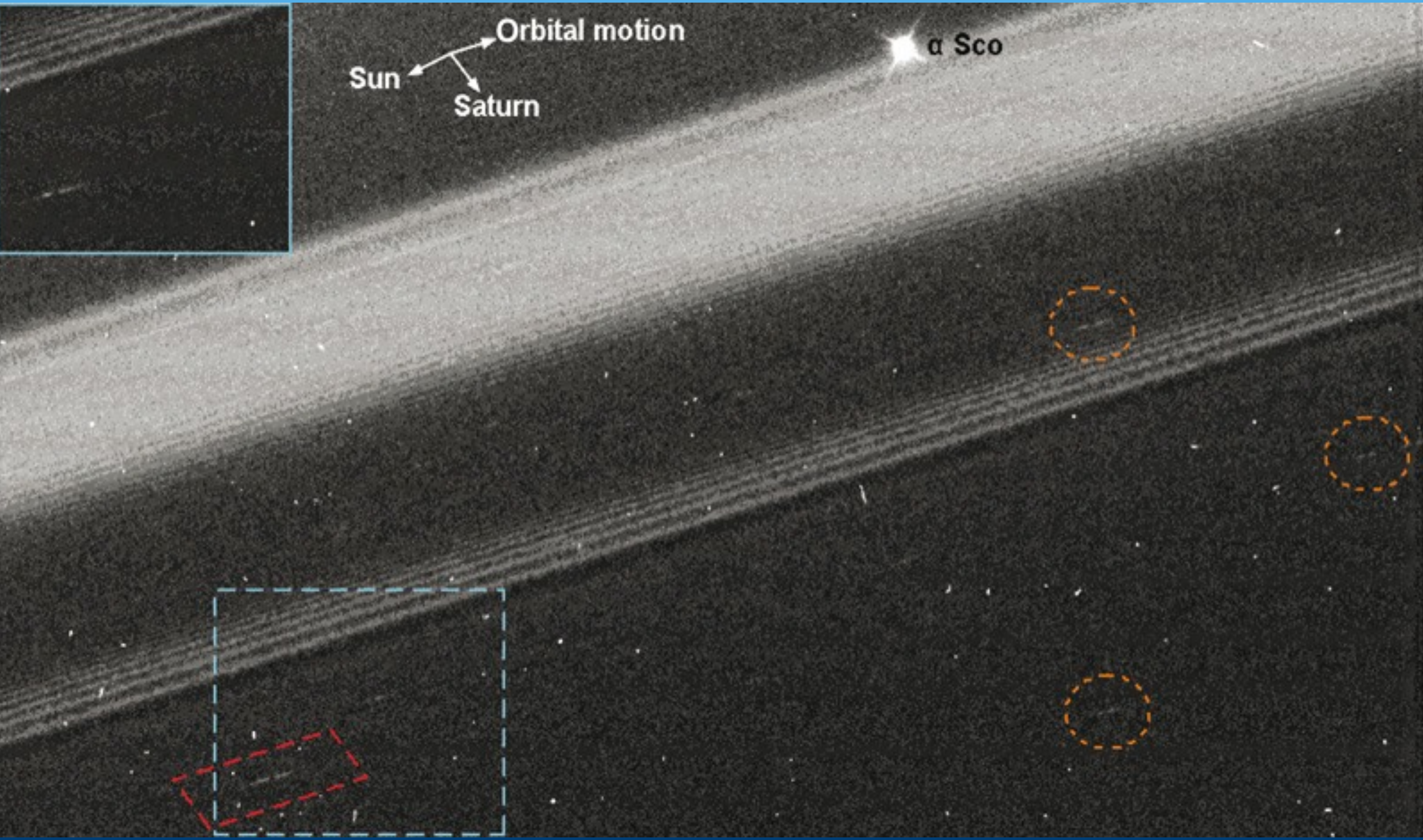
Moonlets in Saturn's Rings

Cassini spacecraft



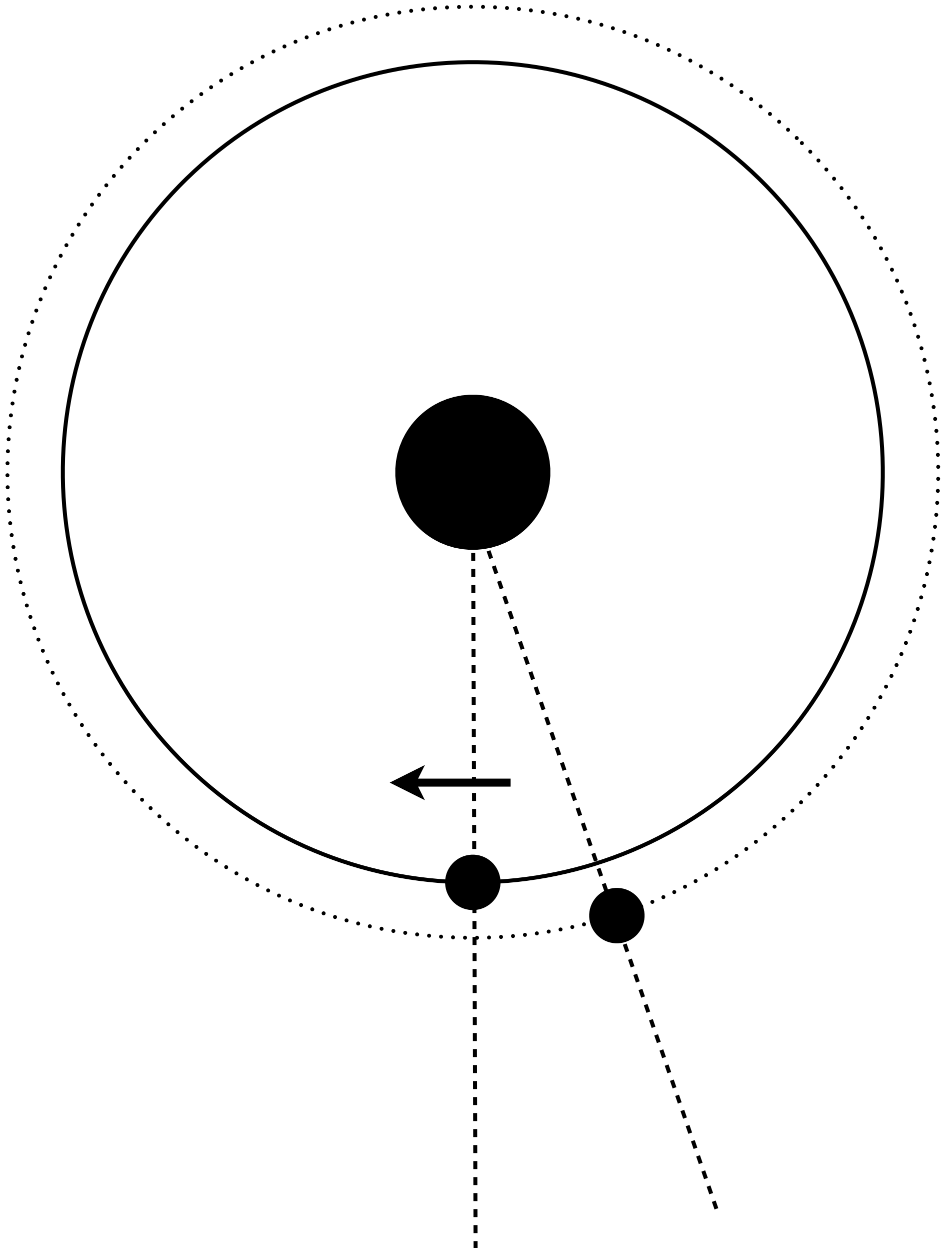
NASA/JPL/Space Science Institute

Propeller structures in A-ring



Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006

Longitude residual



Mean motion [rad/s]

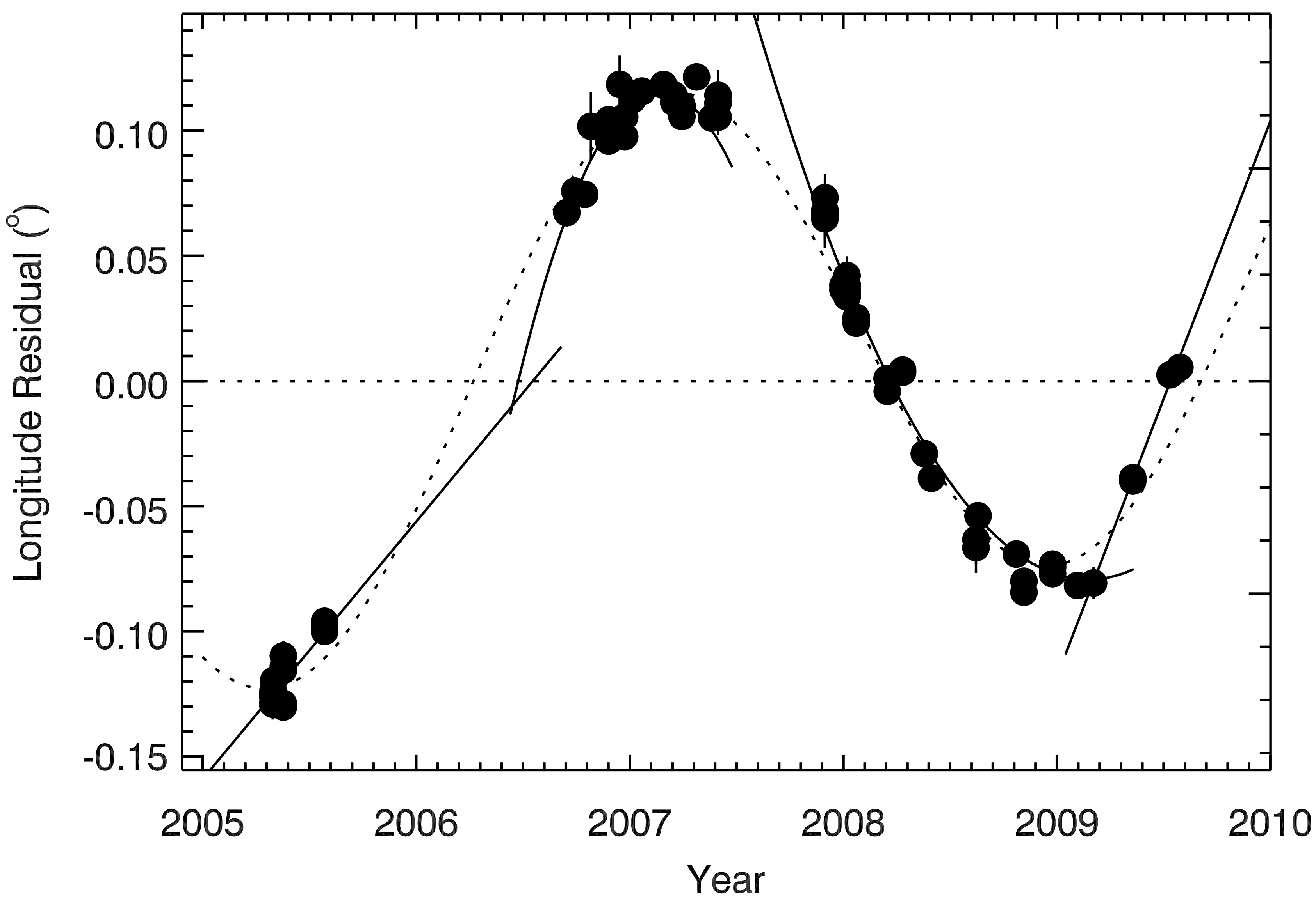
$$n = \sqrt{\frac{GM}{a^3}}$$

Mean longitude [rad]

$$\lambda = n\, t$$

$$\lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t'))\, dt' - \underbrace{\int_0^t n_0\, dt'}_{n_0 t}$$

Observational evidence of non-Keplerian motion



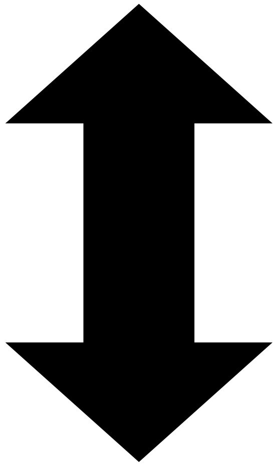
Tiscareno et al. 2010

Random walk

Analytic model

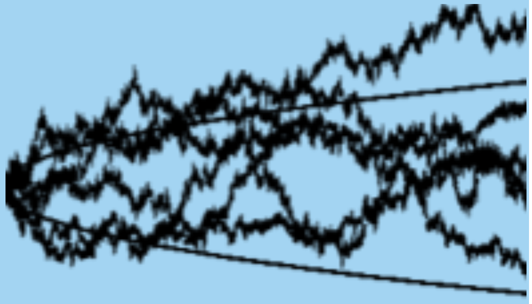
Describing evolution in a statistical manner
Partly based on Rein & Papaloizou 2009

$$\Delta a = \sqrt{4\frac{Dt}{n^2}}$$
$$\Delta e = \sqrt{2.5\frac{\gamma Dt}{n^2 a^2}}$$

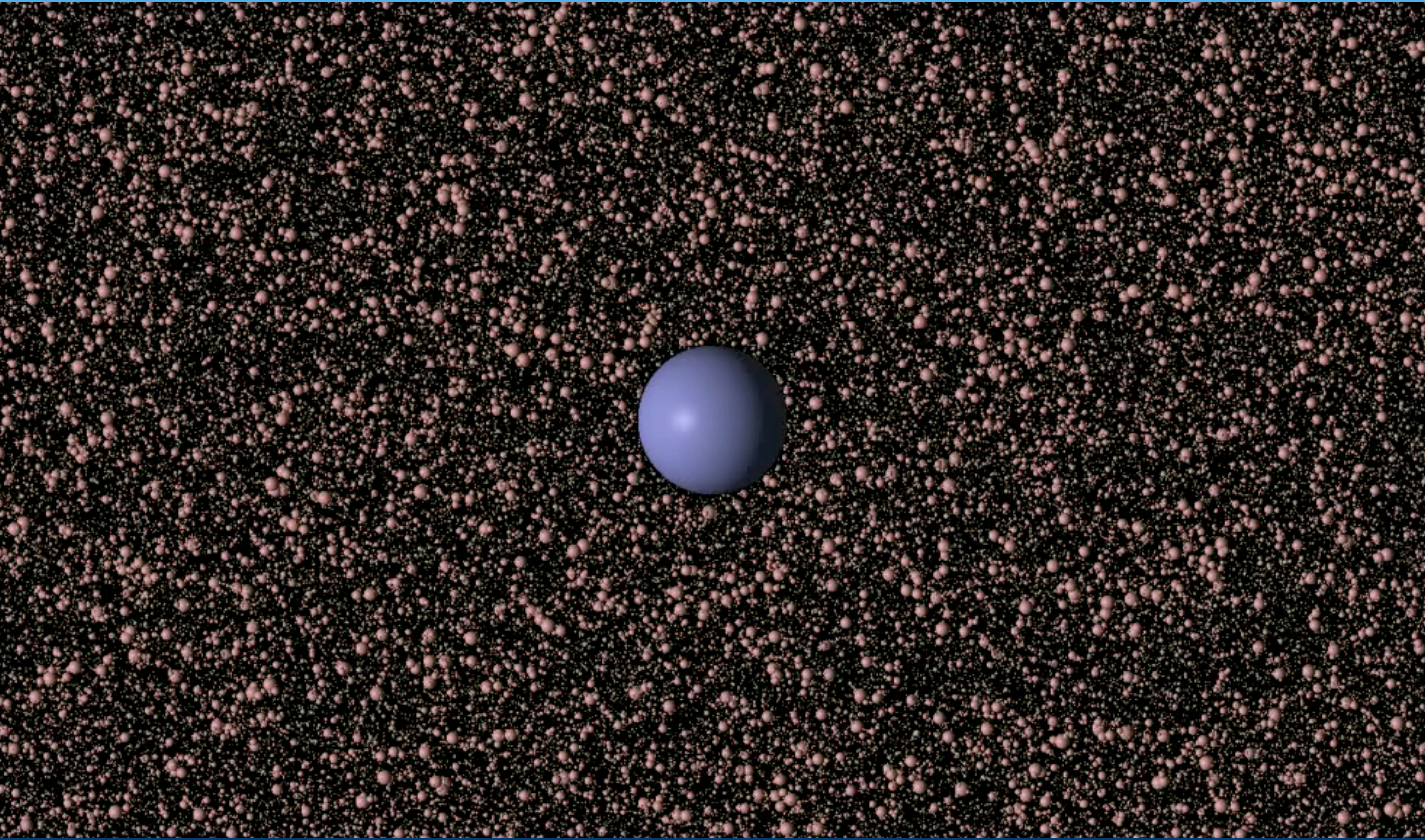


N-body simulations

Measuring random forces or integrating moonlet directly
Crida et al 2010, Rein & Papaloizou 2010

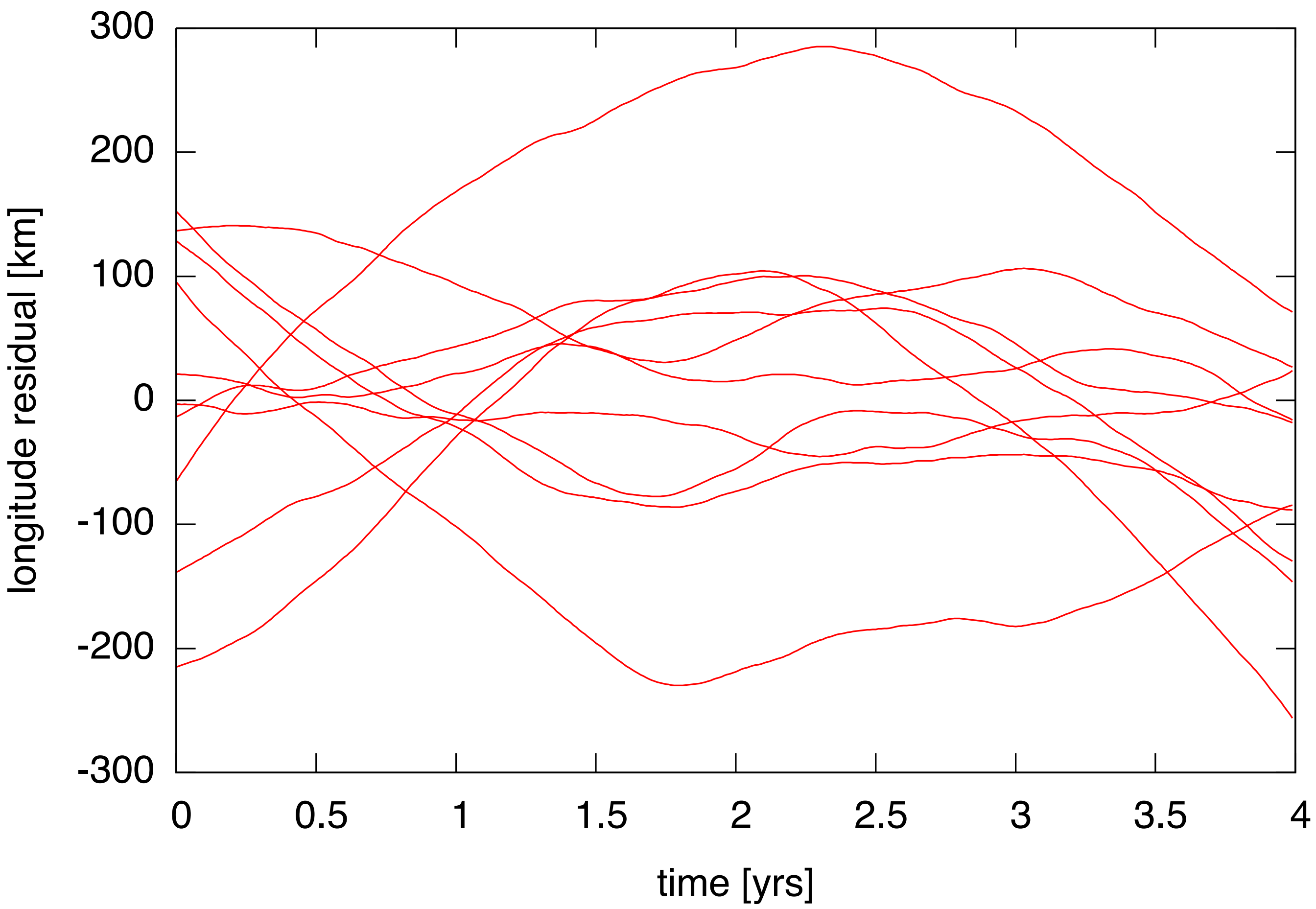


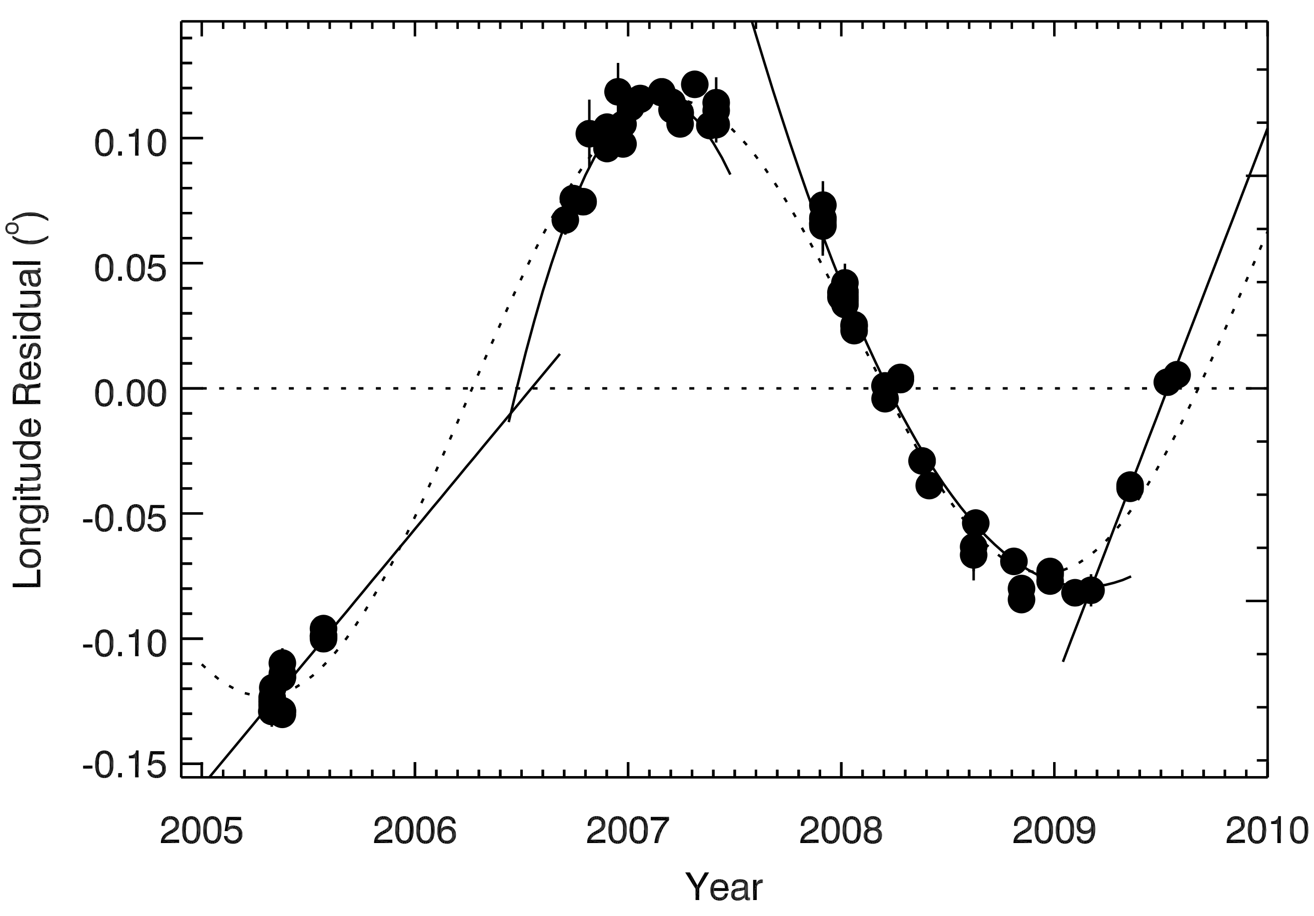
Random walk



REBOUND code, Rein & Papaloizou 2010, Crida et al 2010

Work in progress: a statistical measure





Take home message III

Saturn's rings

=

small scale version of

a proto-planetary disc

REBOUND

A new open source collisional N-body code

Numerical Integrators

- We want to integrate the equations of motions of a particle

$$\dot{x} = v$$
$$\dot{v} = a(x, v)$$

- For example, gravitational potential

$$a(x) = -\nabla\Phi(x)$$

- In physics, these can usually be derived from a Hamiltonian

$$H = \frac{1}{2}p^2 + \Phi(x)$$

- Symmetries of the Hamiltonian correspond to conserved quantities

Numerical Integrators

- Discretization

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= a(x,v) \end{aligned} \longrightarrow \begin{aligned} \Delta x &= v\,\Delta t \\ \Delta v &= a(x,v)\,\Delta t \end{aligned}$$

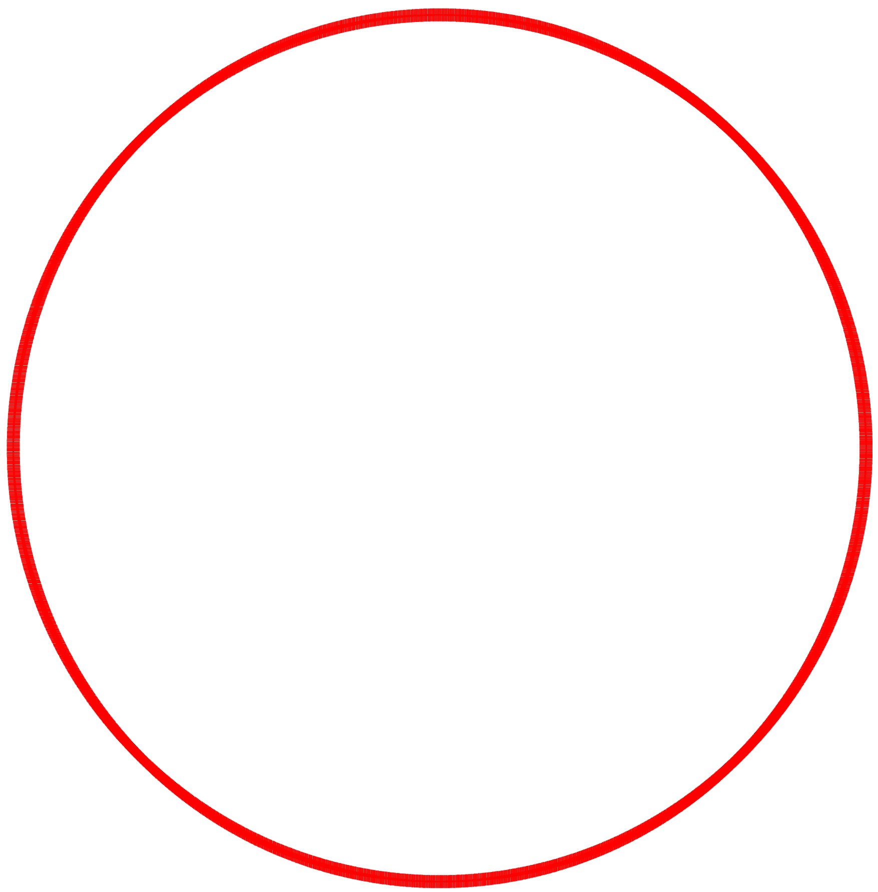
- Hamiltonian

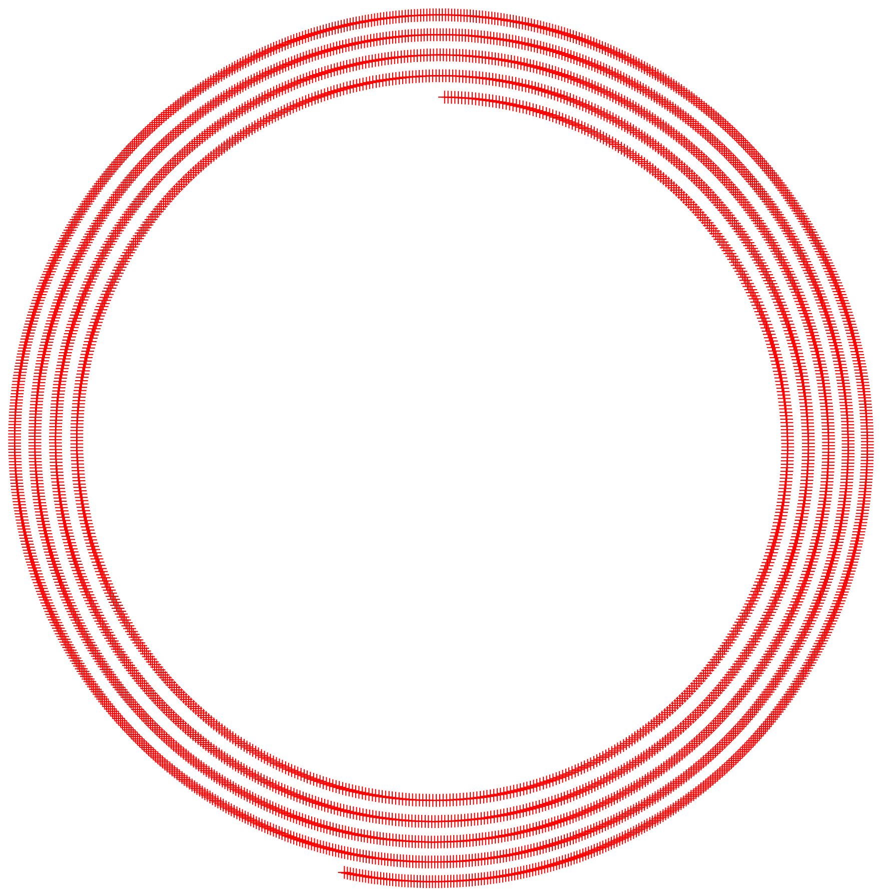
$$H = \frac{1}{2}p^2 + \Phi(x) \longrightarrow \quad ?$$

- The system is governed by a 'discretized Hamiltonian', if and only if the integration scheme is symplectic.

- Why does it matter?

Symplectic vs non symplectic integrators





Mixed variable integrators

- So far: symplectic integrators are great.
- How can it be even better?
- We can split the Hamiltonian:

$$H = H_0 + \epsilon\, H_{\mathrm{pert}}$$

Integrate particle exactly with dominant Hamiltonian

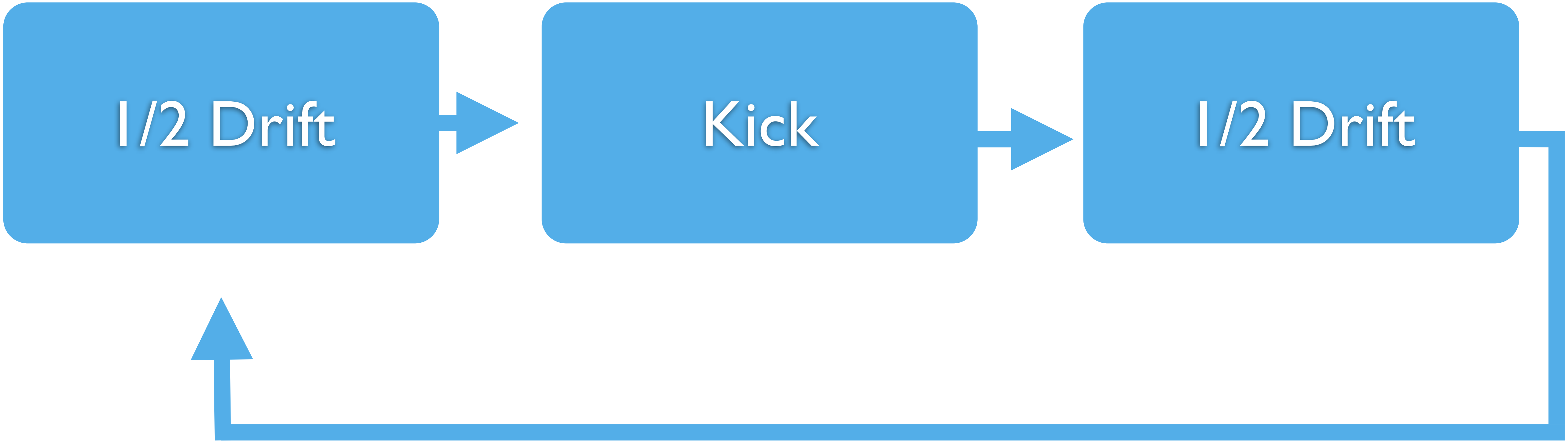
Integrate particle exactly under perturbation Hamiltonian

- Switch back and forth between different Hamiltonians
- Often uses different variables for different parts
- Then:

$$\mathrm{Error} = \epsilon\, (\Delta t)^{p+1}\ [H_0, H_{\mathrm{pert}}]$$

Example: Leap-Frog

$$H = \underbrace{\frac{1}{2}p^2}_{\text{Drift}} + \underbrace{\Phi(x)}_{\text{Kick}}$$

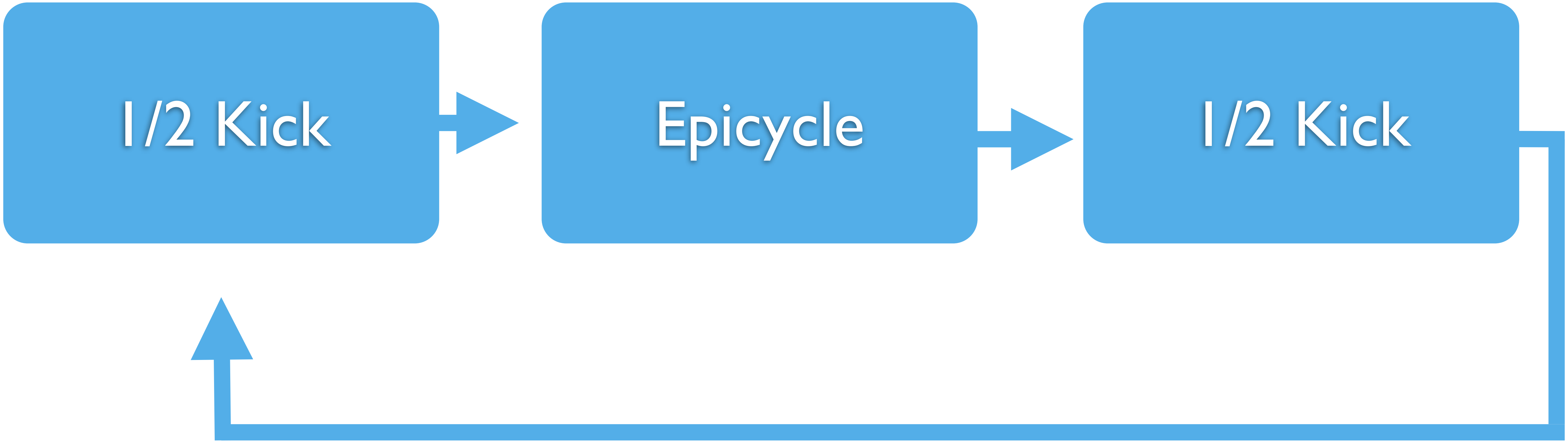


Example: SWIFT/MERCURY

$$H = \underbrace{\frac{1}{2}p^2 + \Phi_{\mathrm{Kepler}}(x)}_{\text{Kepler}} + \underbrace{\Phi_{\mathrm{Other}}(x)}_{\text{Kick}}$$

Example: Symplectic Epicycle Integrator

$$H = \underbrace{\frac{1}{2}p^2 + \Omega(p \times r)e_z + \frac{1}{2}\Omega^2\left[r^2 - 3(r \cdot e_x)^2\right]}_{\text{Epicycle}} + \underbrace{\Phi(r)}_{\text{Kick}}$$

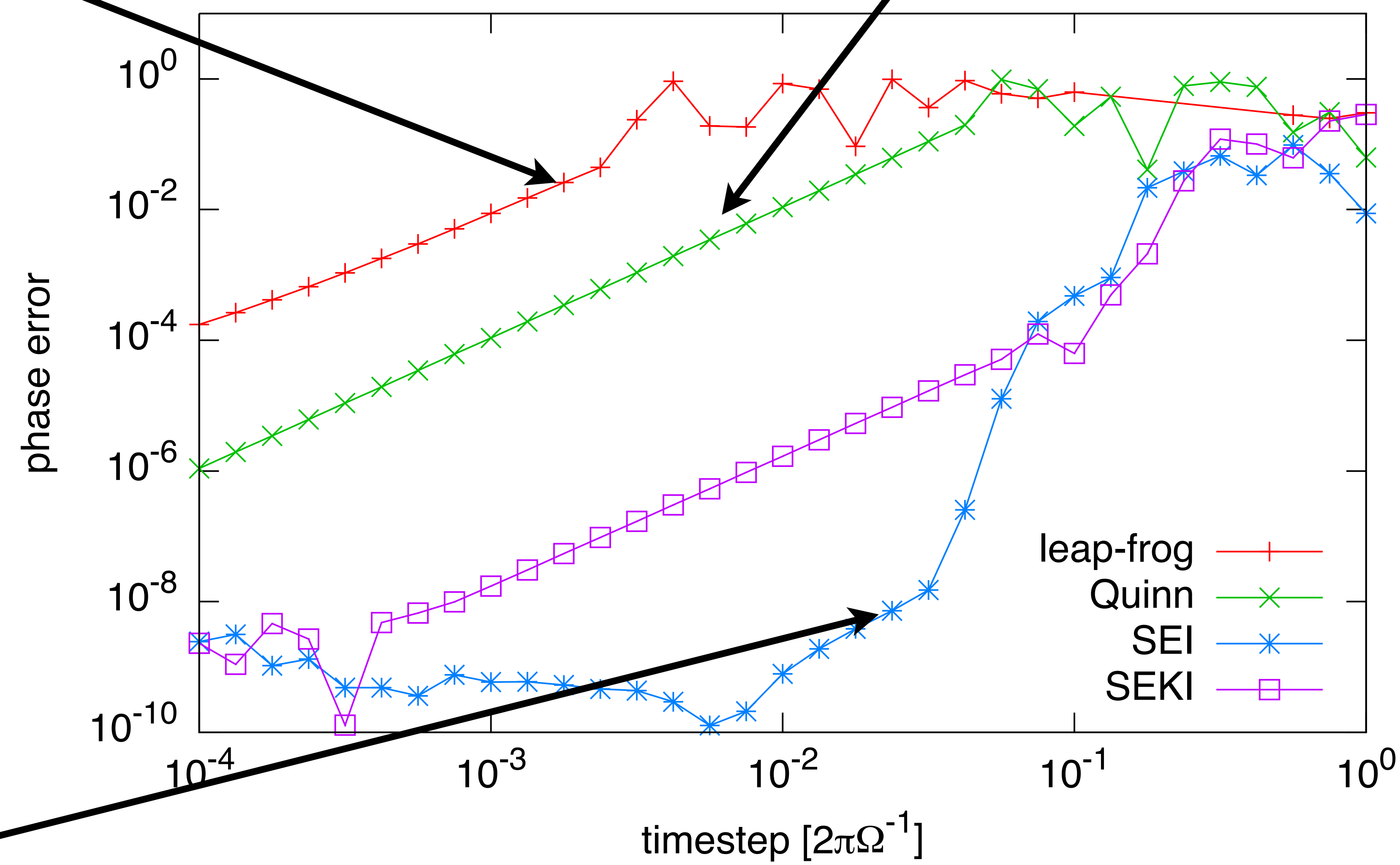


Rein & Tremaine 2011

10 Orders of magnitude better!

non-symplectic

symplectic



mixed variable, symplectic

Take home message IV

symplectic integrators

=

awesome

REBOUND

- Multi-purpose N-body code
- Optimized for collisional dynamics
- Code description paper recently accepted by A&A
- Written in C, open source
- Freely available at http://github.com/hannorein/rebound


Astronomy & Astrophysics manuscript no. paper
October 20, 2011

REBOUND: An open-source multi-purpose N-body code for collisional dynamics

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Submitted: 13 September 2011

ABSTRACT

REBOUND is a new multi-purpose N-body code which is freely available under an open-source license. It was designed for collisional dynamics such as planetary rings but can also solve the classical N-body problem. It is highly modular and can be customized easily to work on a wide variety of different problems in astrophysics and beyond.

REBOUND comes with three symplectic integrators: leap-frog, the symplectic epicycle integrator (SEI) and a Wisdom-Holman mapping (WH). It supports open, periodic and shearing-sheet boundary conditions. REBOUND can use a Barnes-Hut tree to calculate both self-gravity and collisions. These modules are fully parallelized with MPI as well as OpenMP. The former makes use of a static domain decomposition and a distributed essential tree. Two new collision detection modules based on a plane-sweep algorithm are also implemented. The performance of the plane-sweep algorithm is superior to a tree code for simulations in which one dimension is much longer than the other two and in simulations which are quasi-two dimensional with less than one million particles.

In this work, we discuss the different algorithms implemented in REBOUND, the philosophy behind the code's structure as well as implementation specific details of the different modules. We present results of accuracy and scaling tests which show that the code can run efficiently on both desktop machines and large computing clusters.

Key words. Methods: numerical – Planets and satellites: rings – Proto-planetary disks

REBOUND modules

Geometry
- Open boundary conditions
- Periodic boundary conditions
- Shearing sheet / Hill's approximation

Integrators
- Leap frog
- Symplectic Epicycle integrator (SEI)
- Wisdom-Holman mapping (WH)

Gravity
- Direct summation, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- FFT method, $O(N \log(N))$

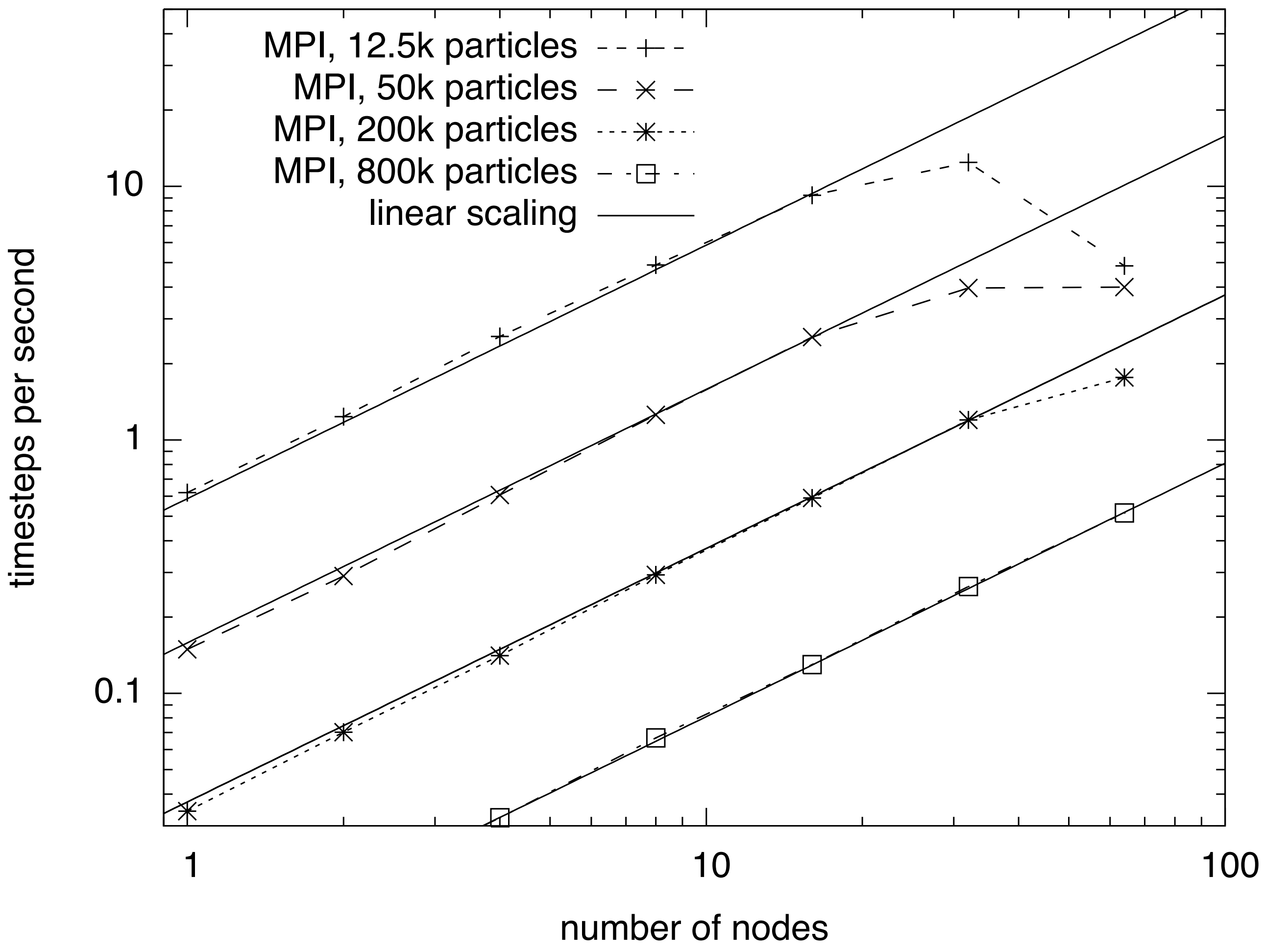
Collision detection
- Direct nearest neighbor search, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- Plane sweep algorithm, $O(N)$ or $O(N^2)$

REBOUND

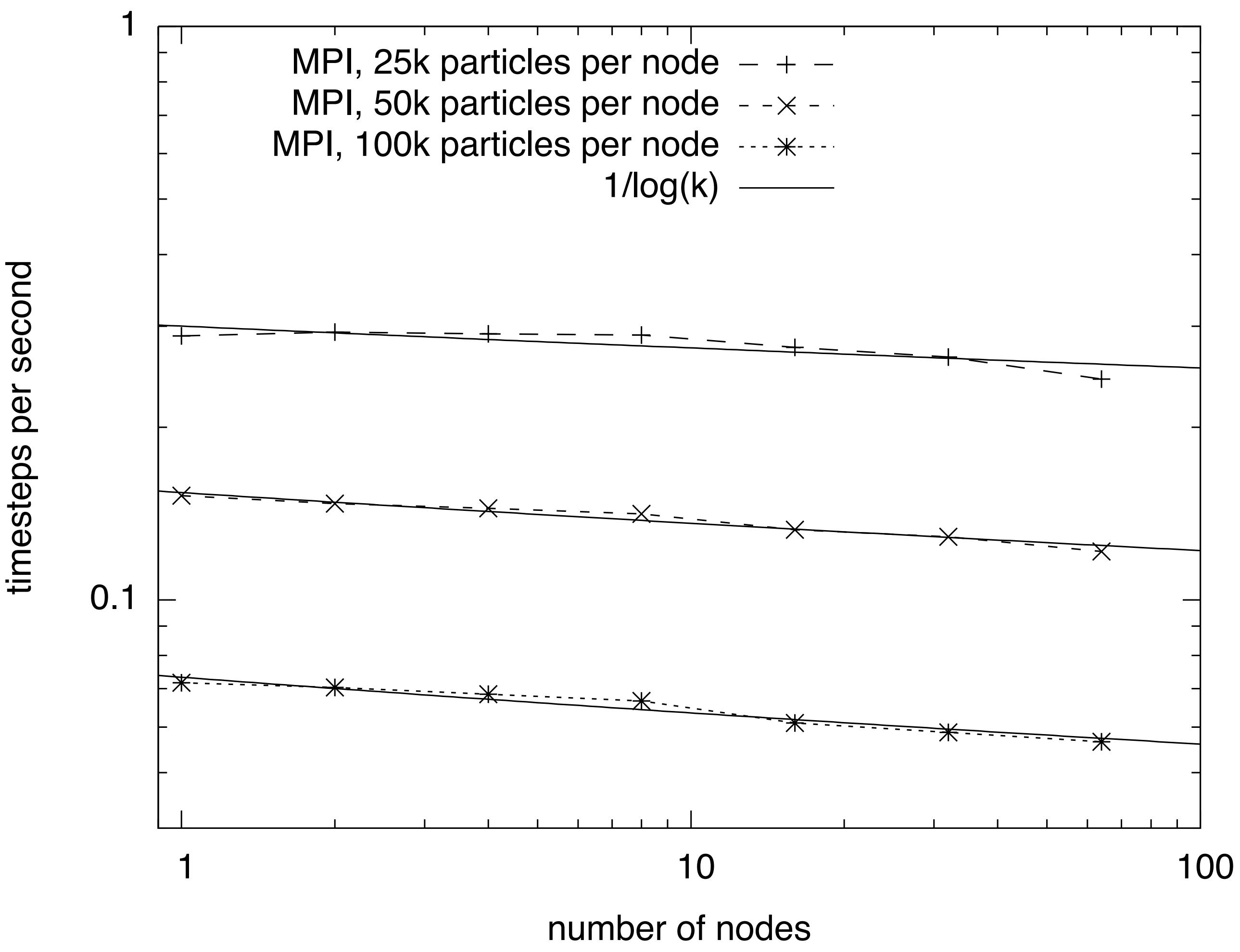
DEMO

REBOUND scalings using a tree

strong



weak



Take home message IIV

Download REBOUND

Conclusions

Conclusions

Resonances and multi-planetary systems

Multi-planetary system provide insight in otherwise unobservable formation phase

GJ876	formed in the presence of a disc and dissipative forces
HD128311	formed in a turbulent disc
HD45364	formed in a massive disc
HD200964	did not form at all

Moonlets in Saturn's rings

Small scale version of the proto-planetary disc
Random walk can be directly observed
Caused by collisions and gravitational wakes

REBOUND

N-body code, optimized for collisional dynamics, uses symplectic integrators
Open source, freely available, very modular and easy to use
http://github.com/hannorein/rebound